1. Introduction

Recently, many countries have moved or are in the process of moving forward to establish full currency convertibility, believing that it would improve financial efficiency by allowing residents to hedge foreign exchange risk. [1] This paper examines whether full currency convertibility does improve welfare, especially when markets are incomplete. More specifically, this paper explores the welfare property of opening trade in country-specific fiat currencies which are fully convertible in the sense that no country imposes any restrictions on the making of payments and transfers for current international transactions. This paper considers an overlapping-generations economy buffeted by stochastic shocks. Incompleteness of markets arises in this paper because it is too costly to establish a complete set of contingency markets for all possible states of realization. This paper demonstrates that free trade in convertible currencies is not necessarily Pareto optimal and its welfare properties depend on the source of uncertainty.

This paper posits two kinds of uncertainty. In the first case, the source of uncertainty is purely monetary. Monetary policy is implemented through uncertain monetary transfers to the old generation in each country. In the second case, the source of uncertainty is purely real. In each case, we compare steady-state equilibria under the Laissez-Faire (LF) regime to those under the portfolio autarky (PA) regime. Under the LF regime agents are allowed to carry both home and foreign currencies. Under the PA regime portfolio choice is restricted such that the residents of home country cannot carry the foreign country's currency between periods.

From the steady-state analysis the following results are obtained. First, in the presence of monetary uncertainty the LF regime is not Pareto superior to the PA regime. The young generation of at least one country must be worse off in the LF regime. If the distributions of the two countries' transfer variables are not too different, it is very likely that all members of the young generation are worse off in the LF regime. In such a case, if the welfare of the current young is compared with that of future generations, then the LF regime is inferior to the PA regime. Under reasonable restrictions, we find that the production of the worse off country rises. Second, in the presence of real uncertainty, currency trading may make all members of the young generation better off and production may increase.

The possibility of deteriorating welfare with monetary uncertainty results from incomplete

[1] See, e. g., Galbis (1996) and Kyei and Yoshimura (1996).

markets. In the absence of a complete set of contingency markets, markets are required to provide more than one service: allocation of goods and allocation of risks. Opening a new market may distort the risk-sharing mechanism so that the market could transmit the risks generated by a foreign country. Agents in this economy clearly have an incentive to trade claims to the uncertain money transfers, since it will allow them complete hedging. In our model there is a continuum of possible states of monetary transfers, and high transactions costs will prevent the existence of a complete set of such contingency markets. Consequently, in the LF regime with a flexible exchange rate system, traders must face the foreign exchange risk that does not exist in the PA regime. This possibility of contagion makes some traders worse off in the LF regime.[2] On the other hand, with real uncertainty, traders in the PA regime face uncertainty in population growth. Currency trading may improve welfare as risks in the two countries could be pooled.

This paper then examines the issue of exchange rate indeterminacy. With stochastic monetary transfers, two currencies become imperfect substitutes, and thus the exchange rate is determinate. However, with real uncertainty, two currencies are perfect substitutes in the LF regime. Without stochastic monetary transfers, there is no reason to distinguish one type of currency from the other. In this case, the exchange rate indeterminacy results with purely real uncertainty as in Kareken and Wallace (1981).

This paper is organized as follows. Section 2 describes the model in an overlapping generations framework. Section 3 analyzes the economy with monetary uncertainty. Section 4 examines the economy with real uncertainty. Section 5 concludes the paper.

2. Model

There are two countries, indexed by j = 1, 2. At the beginning of period t, $q_j(t)$ of identical individuals are born in country j. There are country-specific fiat monies with the following money supply rules:

$$M_j(t+1) = x_j(t+1)M_j(t) \tag{1}$$

where $M_j(t)$ is the money stock of country j in the period t and $x_j(t+1)$ is an independently and

[2] Recently Calvo and Reinhart (1996) provide an empirical evidence on contagion effects involving capital flows. This paper provides a theoretical explanation on how foreign exchange markets could transmit risks across the border.

identically distributed random variable with mean one. Country j 's money supply rule in period t is implemented through an equal monetary transfer to each member of the old generation in period t in country j at the beginning of period t.

The young generation starts with a fixed endowment of leisure and the old with currencies. Facing uncertainties in the rates of return on country-specific currencies, members of the young generation have two optimization problems: portfolio choice and labor supply decision. With free trade in convertible currencies, the young can purchase either domestic or foreign currencies. For simplicity, we assume that consumption occurs only in old age while production takes place solely in youth. The production technology used by the young generation to transform labor into output can be described by $y = l$ where l is labor and y is internationally traded, non-storable output. In order to finance consumption in old age, members of the young generation must sell their output to the old generation to acquire either domestic or foreign currencies. The old simply exchange their holdings of convertible currencies for goods. It is assumed that the law of one price holds in every period:

$$P_1(t) = e(t)P_2(t) \,, \tag{2}$$

where $P_j(t)$ is the price of the consumption good in country j's currency and $e(t)$ is the perfectly flexible exchange rate in period t. The period t budget constraint of member h of generation t in country 1 is

$$P_1(t)\, y_1^h(t) = m_1^h(t) + e(t) m_2^h(t) \tag{3}$$

For convenience, the following institutional features are postulated. Before shocks are realized, a young agent must decide what fractions of his/her nominal income to be allocated in domestic and foreign currencies. These fractions are affected by the distributional characteristics of domestic and foreign, monetary and real shocks. Since all shocks are independently and identically distributed, the share of nominal income to be allocated in foreign (or domestic) assets does not depend on currently realized shocks. We let the proportion of the total savings invested in country 2' s currency by a country 1' s young agent be

$$\alpha_1^h(t) = \frac{e(t)m_2^h(t)}{P_1(t)y_1^h(t)}, \quad 0 \le \alpha_1^h(t) \le 1 \tag{4}$$

where $y_1^h(t)$ is the level of output produced by the agent, $m_j^h(t)$ is the amount of country *j*'s currency purchased, and $\alpha_1^h(t)$ is the proportion of total savings invested in country 2's currency. The agent's consumption in the old age becomes

$$P_1(t+1)c_1^h(t+1) = m_1^h(t) + e(t+1)m_2^h(t) + (x_1(t+1)-1)\frac{M_1(t)}{\theta_1(t)} \tag{5}$$

The third term of the RHS is the stochastic monetary transfer in country 1, realized in the beginning of period t+1. From (3), (4) and (5), the agent's life-time budget constraint becomes

$$c_1^h(t+1) = [\{1-\alpha_1^h(t)\}\frac{P_1(t)}{P_1(t+1)} + \alpha_1^h(t)\frac{P_2(t)}{P_2(t+1)}]y_1^h(t+1) + (x_1(t+1)-1)\frac{M_1(t)}{\theta_1(t)P_1(t+1)} \tag{6}$$

Similar budget constraints can also be obtained for a member of generation t in country 2. From now on the superscript will be suppressed, since all members of generation t in country j are identical. With our notations, the portfolio autarky (PA) regime can be characterized by $\alpha_1 = \alpha_2 = 0$ for all *t*. The equilibrium condition for money market of country 1 becomes

$$\frac{M_1(t)}{P_1(t)} = \{1-\alpha_1(t)\}y_1(t)\theta_1(t) + \alpha_2(t)y_2(t)\theta_2(t) \tag{7}$$

Each member of generation t is assumed to have a separable and concave utility function

$$\begin{aligned} &u_t(c(t+1), l(t)) = \mathrm{E}[\mathrm{U}(c(t+1))] - \mathrm{V}(l(t)) \\ &\mathrm{U}'(0) > \mathrm{V}'(0), \mathrm{U}' > 0, \mathrm{V}' > 0, \mathrm{U}'' < 0, \mathrm{V}'' < 0 \end{aligned} \tag{8}$$

The inequality $\mathrm{U}'(0) > \mathrm{V}'(0)$ guarantees the existence of a monetary equilibrium. The assumption of separability is sufficient for the normality of future consumption and current leisure. In the next two sections we examine the cases where the source of uncertainty is either purely monetary or purely real.

3. Purely Monetary Uncertainty

We now discuss the case where the money supply in each country is stochastic while the size of the population in each country is fixed. Without loss of generality, we assume

$$\boldsymbol{q}_1(t) = \boldsymbol{q}_2(t) = 1 \tag{9}$$

for all t. Substitution of (7) and (9) into (6) yields the agent's life-time budget constraint:

$$c_1(t+1) = [\{1 - \boldsymbol{a}_1(t)\}x_1(t+1)\frac{P_1(t)}{P_1(t+1)} + \boldsymbol{a}_1(t)\frac{P_2(t)}{P_2(t+1)}]y_1(t) + \{x_1(t+1) - 1\}\frac{P_1(t)}{P_1(t+1)}\boldsymbol{a}_2(t)y_2(t) \tag{10}$$

Note that the stochastic monetary transfer makes the real rates of return on country-specific fiat monies different.[3] To a member of generation t of country 1 under the LF regime, the rate of return on country 1's money is $x_1(t+1)\frac{P_1(t)}{P_1(t+1)}$ while that on country 2's money is $\frac{P_2(t)}{P_2(t+1)}$.

To study the welfare properties in the steady-state, we restrict our analysis to steady-state equilibria in which the young generations' decision problems have the stationary solution. In the steady state,

$$y_j(t) = y_j \tag{11}$$

Substituting (2) and (11) into (4), we see that $\boldsymbol{a}_1$ is essentially determined by an agent's real demand for foreign currency. Since the agent must decide the fraction of his/her income allocated in domestic and foreign currency prior to any realizations of current domestic and foreign shocks which are independently and identically distributed, $\boldsymbol{a}_1$ is constant as long as the characteristics of the probability distribution functions governing the shocks do not change. Now from (7) and (11),

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3 In case of lump-sum transfer to a member of generation t the real rate of return on country j's money is $P_j(t)/P_j(t+1)$ regardless of its national identity.

the steady state levels of the two countries' real balances are constant. Thus, the money stock of each country is growing at the same rate as the price level of that country:

$$\frac{P_j(t)}{P_j(t+1)} = \frac{1}{x_j(t+1)} \tag{12}$$

From now on time variables are suppressed for brevity. Instead, future variables are denoted by a prime ('). Also we call country 1 the home country and country 2 the foreign country. In the steady state under the LF regime, a member of generation *t* in home country solves the following problem:

$$\underset{a1,l1}{\text{maximize}} \quad \mathrm{EU}[(c_1')] - \mathrm{V}(l_1) \tag{13a}$$

$$\text{subject to } \; y_1 = l_1, \tag{13b}$$

$$c_1' = y_1 + \boldsymbol{a}_1 y_1 (\frac{1}{x_2'} - 1) - \boldsymbol{a}_2 y_2 (\frac{1}{x_1'} - 1) \tag{13c}$$

where c'_1 is the future consumption under the LF regime. Equation (13c) is derived from (10). Assuming an interior solution, the portfolio choice $\tilde{\boldsymbol{a}}_1$, optimal output $\tilde{y}_1$ and optimal consumption $\tilde{c}_1$ under the LF regime must satisfy the following first order conditions:

$$\mathrm{E}[\mathrm{U}'(\tilde{c}'_1)\{\tilde{y}_1(\frac{1}{x'_2} - 1)\}] = 0 \tag{14a}$$

$$\mathrm{E}[\mathrm{U}'(\tilde{c}'_1)\{1 + \tilde{\boldsymbol{a}}_1(\frac{1}{x'_2} - 1)\}] - \mathrm{V}'(\tilde{y}_1) = 0 \tag{14b}$$

Equations (14a-b) can be rewritten as

$$\mathrm{E}[\mathrm{U}'(\tilde{c}'_1)(\frac{1}{x'_2} - 1)] = 0 \tag{15a}$$

$$\mathrm{E}[\mathrm{U}'(\tilde{c}'_1) = \mathrm{V}'(\tilde{y}_1) \tag{15b}$$

Note that a similar set of optimality conditions for $\tilde{\boldsymbol{a}}_2$, $\tilde{y}_2$, $\tilde{c}'_2$ can be written for the young generation in country 2. In order to derive the optimal foreign currency demand, we approximate

$U'(\tilde{c}'_1)$ using a first-order Taylor series expansion around $\frac{1}{x'_1}=1$ and $\frac{1}{x'_2}=1$ by assuming "small risks" (see Arrow (1971)):[4]

$$U'(\tilde{c}'_1) \approx U'(\tilde{y}_1) + (\frac{1}{x'_2}-1)U''(\tilde{y}_1)\tilde{\boldsymbol{a}}_1\tilde{y}_1 - (\frac{1}{x'_1}-1)U''(\tilde{y}_1)\tilde{\boldsymbol{a}}_2\tilde{y}_2 \qquad (16)$$

Substitution (16) into (15a) yields the agent's optimal foreign currency demand:[5]

$$\tilde{\boldsymbol{a}}_1\tilde{y}_1 = -\frac{U'(\tilde{y}_1)}{U''(\tilde{y}_1)}\frac{E[\frac{1}{x'_2}-1]}{E[(\frac{1}{x'_2}-1)^2]} = \frac{1}{R_A(\tilde{y}_1)}\frac{E[\frac{1}{x'_2}-1]}{E[(\frac{1}{x'_2}-1)^2]} \qquad (17a)$$

where $R_A(\tilde{y}_1)$ is the degree of absolute risk aversion measured at $y_1=\tilde{y}_1$. That is, $R_A(\tilde{y}_1) = -\frac{U''(\tilde{y}_1)}{U'(\tilde{y}_1)}$. By Jensen`s inequality, $E[\frac{1}{x'_2}-1]>0$. Thus, the optimal foreign currency demand is positive. Substitution (16) into (15b) yields the optimal production:

$$E[U'(\tilde{c}_1')] = U'(\tilde{y}_1)\{1-\Delta_2+\Delta_1\frac{R_A(\tilde{y}_1)}{R_A(\tilde{y}_2)}\} = V'(\tilde{y}_1). \qquad (17b)$$

where $\Delta_j = (E[\frac{1}{x_j'}-1])^2 / E[(\frac{1}{x_j'}-1)^2]$. The welfare level attained under the LF regime is given by $\tilde{W}_j = E[U(\tilde{c}_j')] - V(\tilde{y}_j)$ for j = 1, 2. Note that the degree of absolute risk aversion, R_A, is not required to be constant for our results.

We will compare the steady-state equilibria under the PA regime with those under the LF

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4 $U'(c_1') = U'(y_1 + \boldsymbol{a}_1 y_1(\frac{1}{x_2'}-1) - \boldsymbol{a}_2 y_2(\frac{1}{x_1'}-1))$

$\approx U'(y_1) + (\frac{1}{x_2'}-1)U''(y_1)\boldsymbol{a}_1 y_1 - (\frac{1}{x_1'}-1)U''(y_1)\boldsymbol{a}_2 y_2$.

5 $E[(\frac{1}{x_1'}-1)(\frac{1}{x_2'}-1)] \approx 0$ for small risks.

regime. While the optimal output $\tilde{y}_1$ under the LF regime must satisfy (15), the optimal output $\tilde{y}_1$ under the PA regime (which means $\boldsymbol{a}=0$) can be characterized by the following first-order condition:

$$\mathrm{U}'(\hat{y}_1)=\mathrm{V}'(\hat{y}_1) \tag{18}$$

The welfare level attained with $\hat{y}_j$ under the PA regime is given by $\hat{W}_j=\mathrm{U}(\hat{y}_j)-\mathrm{V}(\hat{y}_j)$ for $j=1$, 2. Now, we are ready to compare welfare levels for steady-state equilibria under different regimes.

<u>Proposition 1</u>. In the presence of monetary uncertainty and small risks, currency trade in the Laissez-Faire (LF) regime makes the young generation of at least one country worse off than in the Portfolio Autarky (PA) regime. Under the LF regime the production of the country whose young generation is worse off is larger than or equal to that under the PA regime.

<u>Proof</u>: Assuming small risks as in Arrow (1971), we approximate $\mathrm{U}(c_1')$ using a second-order Taylor expansion around $\frac{1}{x_1'}=1$ and $\frac{1}{x_2'}=1$:

$$\begin{aligned}\mathrm{U}(c_1') \approx{}& \mathrm{U}(y_1)+\boldsymbol{a}_1 y_1(\frac{1}{x_2'}-1)\mathrm{U}'(y_1)-\boldsymbol{a}_2 y_2(\frac{1}{x_1'}-1)\mathrm{U}'(y_1)\\ &+\frac{1}{2}(\boldsymbol{a}_1 y_1)^2(\frac{1}{x_2'}-1)^2\,\mathrm{U}''(y_1)-\frac{1}{2}(\boldsymbol{a}_2 y_2)^2(\frac{1}{x_1'}-1)^2\,\mathrm{U}''(y_1)\\ &+\boldsymbol{a}_1 y_1\boldsymbol{a}_2 y_2(\frac{1}{x_1'}-1)(\frac{1}{x_2'}-1)\mathrm{U}''(y_1)\end{aligned} \tag{19}$$

The optimal currency demand and the expected utility of future consumption under the LF regime are obtained:

$$\mathrm{E}[\mathrm{U}(\tilde{c}_1'(\tilde{y}_1))]\approx \mathrm{U}(\tilde{y}_1)+\frac{1}{2}[\frac{\Delta_2}{R_A(\tilde{y}_1)}-\frac{\Delta_1}{R_A(\tilde{y}_2)}-\frac{\Delta_1}{R_A(\tilde{y}_2)}(1+\frac{R_A(\tilde{y}_1)}{R_A(\tilde{y}_2)})]\mathrm{U}'(\tilde{y}_1) \tag{20}$$

where $\Delta_j=\dfrac{[\mathrm{E}(\frac{1}{x_j'}-1)]^2}{\mathrm{E}[(\frac{1}{x_j'}-1)^2]}>0$. If $\dfrac{\Delta_2}{R_A(\tilde{y}_1)}\le\dfrac{\Delta_1}{R_A(\tilde{y}_2)}$, then $\mathrm{E}[\mathrm{U}(\tilde{c}_1'(\tilde{y}_1))]<\mathrm{U}(\tilde{y}_1)$ from (20). Hence,

$\mathrm{E}[\mathrm{U}(\tilde{c}_1'(\tilde{y}_1))] - \mathrm{V}(\tilde{y}_1) < \mathrm{U}(\tilde{y}_1) - \mathrm{V}(\tilde{y}_1)$. Since $\hat{y}_1$ is the optimal choice under the PA regime, $\mathrm{U}(\tilde{y}_1) - \mathrm{V}(\tilde{y}_1) < \mathrm{U}(\hat{y}_1) - \mathrm{V}(\hat{y}_1)$. Thus, $\tilde{\mathrm{W}}_1 = \mathrm{E}[\mathrm{U}(\tilde{c}_1'(\tilde{y}_1))] - \mathrm{V}(\tilde{y}_1) < \mathrm{U}(\hat{y}_1) - \mathrm{V}(\hat{y}_1) = \hat{W}_1$. That is, the welfare level of the home young under the LF regime is strictly less than that under the PA regime. For the case of $\frac{\Delta_2}{R_A(\tilde{y}_1)} \geq \frac{\Delta_1}{R_A(\tilde{y}_2)}$, a similar argument can be used to show that the welfare of the foreign young under the LF regime is lower than that under the PA regime. Since one of the two inequalities must hold, the first part of the proposition is proved.

From (17b) and (18), we have $\mathrm{U}'(\hat{y}_1) - \mathrm{V}'(\hat{y}_1) = 0$ and $\mathrm{U}'(\tilde{y}_1) - \mathrm{V}'(\tilde{y}_1) = U'(\tilde{y}_1)(\frac{\Delta_2}{R_A(\tilde{y}_1)} - \frac{\Delta_1}{R_A(\tilde{y}_2)})R_A(\tilde{y}_1)$. If $\frac{\Delta_2}{R_A(\tilde{y}_1)} \leq \frac{\Delta_1}{R_A(\tilde{y}_2)}$, then $\tilde{W}_1 < \tilde{W}_2$ as shown above and $\mathrm{U}'(\tilde{y}_1) - \mathrm{V}'(\tilde{y}_1) \leq 0$. Since $\mathrm{U}'' - \mathrm{V}'' < 0$, we know that the function $\mathrm{U}' - \mathrm{V}'$ is strictly decreasing in y_1. Given $\mathrm{U}'(\hat{y}_1) - \mathrm{V}'(\hat{y}_1) = 0$ and $\mathrm{U}'(\tilde{y}_1) - \mathrm{V}'(\tilde{y}_1) \leq 0$, we have $\tilde{y}_1 \geq \hat{y}_1$ if $\frac{\Delta_2}{R_A(\tilde{y}_1)} \leq \frac{\Delta_1}{R_A(\tilde{y}_2)}$. A similar argument can be used for the case where $\frac{\Delta_2}{R_A(\tilde{y}_1)} \geq \frac{\Delta_1}{R_A(\tilde{y}_2)}$ to show that $\tilde{W}_1 > \tilde{W}_2$ and $\tilde{y}_1 \leq \hat{y}_1$. This completes the proof for the second part of the proposition. ||

Proposition 1 demonstrates that full currency convertibility never leads to Pareto improvement for any economy characterized by quite general utility functions and probability distribution. In order to understand the implications of this proposition, let us rearrange the expected utility of the future consumption of the home young:

$$\begin{aligned} \mathrm{E}[\mathrm{U}(\tilde{c}_1'(\tilde{y}_1))] \approx{} & \mathrm{U}(\tilde{y}_1) + \boldsymbol{a}_1 y_1 \mathrm{E}(\frac{1}{x'_2} - 1)\mathrm{U}'(\tilde{y}_1) - \tilde{\boldsymbol{a}}_2 \tilde{y}_2 \mathrm{E}(\frac{1}{x'_1} - 1)\mathrm{U}'(\tilde{y}_1) \\ & - \frac{1}{2}\{(\tilde{\boldsymbol{a}}_1 \tilde{y}_1)^2 \mathrm{E}(\frac{1}{x_2'} - 1)^2 - (\tilde{\boldsymbol{a}}_2 \tilde{y}_2)^2 \mathrm{E}(\frac{1}{x_1'} - 1)^2 \\ & + 2(\tilde{\boldsymbol{a}}_1 \tilde{y}_1)(\tilde{\boldsymbol{a}}_2 \tilde{y}_2)\mathrm{E}(\frac{1}{x_1'} - 1)\mathrm{E}(\frac{1}{x_2'} - 1)\} R_A(\tilde{y}_1)\mathrm{U}'(\tilde{y}_1) \end{aligned} \quad (21)$$

The second term of the RHS measures the marginal increase in expected utility due to domestic investment in foreign currency. The third and fourth terms measure the marginal loss of expected utility due to foreign investment in domestic currency. That the second term is positive indicates that if currency trading is allowed, purchasing foreign currency is beneficial. However, portfolio diversification by the foreign young creates external diseconomies, shown as the negative terms in the above equation and (13c). Currency trading transmits foreign uncertainty into the domestic economy, and this reduces the welfare of the home young. The loss from external diseconomies may be larger than the gain. Note that the phenomenon of the foreign exchange market transmitting risks across the border is called contagion (see, e.g., Calvo and Reinhart (1996)). This paper provides one theoretical justification for it.

Proposition 2. In the presence of monetary uncertainty and small risks, there exists a set of parameter values for which the steady-state welfare levels of the young in both countries under the LF regime are strictly lower than those under the PA regime. For this set of parameter values, production in each country under the LF regime is larger than or equal to that under the PA regime.

Proof: By setting $\frac{\Delta_2}{R_A(\tilde{y}_1)} = \frac{\Delta_1}{R_A(\tilde{y}_2)}$, or more generally by requiring the condition

$\{2 + \frac{R_A(\tilde{y}_2)}{R_A(\tilde{y}_1)}\}^{-1} < \frac{\Delta_2}{\Delta_1}\frac{R_A(\tilde{y}_2)}{R_A(\tilde{y}_1)} < 2 + \frac{R_A(\tilde{y}_1)}{R_A(\tilde{y}_2)}$ in Proposition 1, we see that the rest follows. ||

The welfare of any current or future young agents could deteriorate with currency trading. Here Δ_j is strictly positive for nondegenerate random variables. If the distributions governing money growth of two countries are similar (such that $\frac{\Delta_2}{\Delta_1}$ is in a bounded interval), then young agents in both countries are worse off under the LF regime. The result of welfare-deteriorating currency trading is due to incomplete markets. With incomplete risk sharing, traders exchange risks, which do not exist in the PA regime. If currency trading makes all members of the young worst off, then the old must be better off, since steady-state consumption under the LF regime is larger than that under the PA regime. The realization of the transfer variables will determine the actual distribution of consumption between two countries. Note that portfolio choice depends on the distributions governing money growth (see (17)), and the labor supply decision depends on portfolio choice. Thus, any changes in the distribution of money growth rates will affect the steady-state equilibria.[6] This results from the fact that governments distribute monetary transfers only to their own residents.

The equilibrium exchange rate is determined by the law of one price (2) and the money market equilibrium condition (7):

$$e(t) = \frac{M_1(t)}{M_2(t)} \frac{y_2(1-a_2)+y_1 a_1}{y_1(1-a_1)+y_2 a_2} \tag{22}$$

Our result differs from Kareken and Wallace (1981), in which the exchange rate is undetermined. Their indeterminacy result was derived in the absence of monetary uncertainty. With certainty the real rate of return on each country's currency must be the same. Otherwise, the currency with the lower rate of return will collapse. If the rates of return are identical, then two currencies become perfect substitutes and the exchange rate is indeterminate as is the price level in each country. However, as long as there is uncertainty in the transfer variables, then two currencies are not perfect substitutes to risk-averse traders and the equilibrium exchange rate is uniquely determined.[7]

4. Purely Real Uncertainty

We now examine the case where the source of uncertainty is purely real. With real uncertainty the returns on savings are no longer constant even in the PA regime. Thus, unlike the previous case, opening currency trading may generate risk-pooling effects. Real uncertainty is modeled by the following stochastic distribution of the population growth rate:

$$q_j(t+1) = \{1 + w_j(t+1)\} q_j(t), \tag{23}$$

where $w_j(t+1)$ is an independently and identically distributed random variable with mean zero. Again, we assume $x_j(t) = 1$ with probability one for all j and t. Without country-specific transfers, members of generation t in both countries are identical under the LF regime. Therefore, $a_1 = 1 - a_2$

[6] For example, a scale change in the gross growth rate of money stock in only one country ($x = lc$ where l is positive number) will result in a different steady-state equilibrium, although this change has no impact on real variables in portfolio autarky.

[7] Lapan and Enders (1983) also demonstrated the exchange rate determinacy in a model with probabilistic capital controls.

and $y_1 = y_2 = y$. Let $\boldsymbol{a}_1 = \boldsymbol{a}$. Under the LF regime the steady-state rates of return on the two currencies are equal from equation (7):

$$\frac{P_1}{P_1'} = \frac{P_2}{P_2'} = \frac{\boldsymbol{q}_1' + \boldsymbol{q}_2'}{\boldsymbol{q}_1 + \boldsymbol{q}_2} = 1 + \boldsymbol{b}\boldsymbol{w}_1' + (1-\boldsymbol{b})\boldsymbol{w}_2' \qquad (24)$$

where $\boldsymbol{b} = \frac{\boldsymbol{q}_1}{\boldsymbol{q}_1 + \boldsymbol{q}_2}$ (see (23)) and $0 < \boldsymbol{b} < 1$. This implies that two currencies are perfect substitutes under LF regime. Therefore, the portfolio choice variable $\boldsymbol{a}$ is undetermined, and so are the exchange rate and the price level in each country:

$$e = \frac{M_1}{M_2} \frac{\boldsymbol{a}}{1-\boldsymbol{a}} \qquad (25)$$

As in Kareken and Wallace (1981), the level of the exchange rate does not change over time while the rate of return of each currency is stochastic. On the other hand, under the PA regime the steady state rates of return on two currencies differ from each other since

$$\frac{P_j}{P_j'} = \frac{\boldsymbol{q}_j}{\boldsymbol{q}_j'} = 1 + \boldsymbol{w}_j', \text{ for } j = 1, 2.$$

The production decision under the LF regime ($\tilde{y}_1$) and the PA regime ($\hat{y}_1$) are characterized by the following first-order conditions:

$$\mathrm{E}[\mathrm{U}'((\boldsymbol{b}(1+\boldsymbol{w}_1') + (1-\boldsymbol{b})(1+\boldsymbol{w}_2'))\tilde{y}_1)(\boldsymbol{b}(1+\boldsymbol{w}_1') + (1-\boldsymbol{b})(1+\boldsymbol{w}_2')) \mid \boldsymbol{b}] = \mathrm{V}'(\tilde{y}_1) \qquad (26a)$$

$$\mathrm{E}[\mathrm{U}'((1+\boldsymbol{w}_1')\hat{y}_1(1+\boldsymbol{w}_1'))] = \mathrm{V}'(\hat{y}_1) \qquad (26b)$$

The welfare level of the young under the LF regime is:

$$\tilde{W}_1 = \mathrm{E}[\mathrm{U}((1+\boldsymbol{b}\boldsymbol{w}_1' + (1-\boldsymbol{b})\boldsymbol{w}_2')\tilde{y}_1) \mid \boldsymbol{b}] = \mathrm{V}(\tilde{y}_1) \qquad (27)$$

The welfare level of the young under the PA regime is:

$$\tilde{W}_1 = \mathrm{E}[\mathrm{U}((1+w_1')\hat{y}_1)] - \mathrm{V}(\hat{y}_1) \tag{28}$$

Thus, with real uncertainty currency returns are stochastic under the PA regime, so currency trading may pool risks in the two countries and make everyone better off. We now demonstrate that for some choice of utility functions and probability distribution functions, currency trading could lead to Pareto improvement with endogenous production and real uncertainty.

Proposition 3: In the presence of real uncertainty and small risks, there exists a set of parameter values such that currency trading makes young agents of each country better off. Both countries produce more under the LF regime than under the PA regime.

Proof: Using the Taylor series expansion of $\mathrm{U}'(\cdot)$ around $w_1' = w_2' = 0$ and assuming small risks, we can rewrite (26) as follows:

$$\mathrm{U}'(\tilde{y}_1) + \mathrm{U}''(\tilde{y}_1)\tilde{y}_1\{b^2 s_1^{\ 2} + 2b(1-b)s_{12} + (1-b)^2 s_2^2\} = \mathrm{V}'(\tilde{y}_1) \tag{29a}$$

$$\mathrm{U}'(\hat{y}_1) + \mathrm{U}''(\hat{y}_1)\hat{y}_1 s_1^2 = \mathrm{V}'(\hat{y}_1) \tag{29b}$$

where s_j^2 is the variance of w_j' and s_{12} is the covariance of w_1' and w_2'. Let $\Psi = b^2 s_1^{\ 2} + 2b(1-b)s_{12} + (1-b)^2 s_2^2$. Then, Ψ can be strictly less than the smaller of s_1^2 and s_2^2 with appropriate choices of some negative covariance s_{12} and parameter b. Assuming constant relative risk aversion, $-\mathrm{U}''(y)y/\mathrm{U}'(y) = R_R$, we can express (29) as follows :

$$1 - R_R\Psi = \mathrm{V}'(\tilde{y}_1)/\mathrm{U}'(\tilde{y}_1) \tag{30a}$$

$$1 - R_R s_1^{\ 2} = \mathrm{V}'(\hat{y}_1)/\mathrm{U}'(\hat{y}_1) \tag{30b}$$

Note that $\mathrm{V}'(y_1)/\mathrm{U}'(y_1)$ is strictly increasing in y and $\Psi < s_1^{\ 2}$. Hence, $\tilde{y}_1 > \hat{y}_1$. A similar argument can also be used to show $\tilde{y}_1 = \tilde{y}_2 > \hat{y}_2$. Therefore, young agents in each country produce more under the LF regime than under the PA regime.

Using the Taylor series expansion of $U'(\cdot)$ around $w_1' = w_2' = 0$, we can derive the following from (27) , (28), (29), and (30):

$$\tilde{W}_1 = U(\tilde{y}_1) - V(\tilde{y}_1) - \frac{1}{2} R_R U'(\tilde{y}_1)\tilde{y}_1 \Psi \tag{31}$$

$$\hat{W}_1 = U(\hat{y}_1) - V(\hat{y}_1) - \frac{1}{2} R_R U'(\hat{y}_1)\hat{y}_1 s_1^2 \tag{32}$$

To show $\tilde{W}_1 > \hat{W}_1$, we first demonstrate that $U(\tilde{y}_1) - V(\tilde{y}_1) > U(\hat{y}_1) - V(\hat{y}_1)$. Let $\bar{y}$ is determined by $U'(\bar{y}_1) = V'(\bar{y}_1)$. Since $U'' - V'' < 0$ and $U'(0) - V'(0) > 0$, we have: $U'(y) - V'(y) > 0$ if $0 \le y < \bar{y}$; $U'(y) - V'(y) = 0$ if $y = \bar{y}$; and $U'(y) - V'(y) < 0$ if $y > \bar{y}$. Furthermore, $U(\tilde{y}_1) - V(\tilde{y}_1) > 0$ and $U(\hat{y}_1) - V(\hat{y}_1) >$ 0 because $U'' < 0$. Thus, $\hat{y}_1 < \tilde{y}_1 < \bar{y}$ and $U(\tilde{y}_1) - V(\tilde{y}_1) > U(\hat{y}_1) - V(\hat{y}_1)$. Second, we need to show $U'(\tilde{y}_1)\tilde{y}_1 \Psi < U'(\hat{y}_1)\hat{y}_1 s_1^2$, which implies from (30) and (31), $\tilde{W}_1 > \hat{W}_1$. Since $\Psi < s_1^2$, it is sufficient to show $U'(\tilde{y}_1)\tilde{y}_1 < U'(\hat{y}_1)\hat{y}_1$. This inequality holds if $yU'(y)$ is strictly decreasing. Differentiation of $yU'(y)$ yields $U'(y) + yU''(y) = U'(y)[1 - R_R]$, which becomes strictly negative if $R_R > 1$. In this case, $\tilde{W}_1 > \hat{W}_1$. Similarly we can derive parametric restrictions under which $\tilde{W}_2 > \hat{W}_2$. ||

Proposition 3 indicates that it is possible to make all young generations better off with currency trading under real uncertainty. In this case, since output levels are higher in both countries, the old generations are also better off with currency trading. Therefore, currency trading with real uncertainty could lead to Pareto improvement. We summarize our results as follows: with monetary uncertainty, it is not possible to achieve Pareto improvement with currency trading; with real uncertainty, it is possible.

5. Concluding Remarks

This paper explores the welfare implications of fully convertible currency trading. When the source of uncertainty is purely monetary, currency trading must make the young generation of at least one country worse off. The results in this paper do not support the general perception that exchange rate risk can be diversified in an efficient way in the foreign exchange market, especially when markets are incomplete. That is, the foreign exchange market could transmit the risks generated from foreign countries, which may distort the allocation mechanism of the price system. In such a case, opening another market such as the forward exchange market while maintaining the incompleteness of the market structure may not improve the welfare of agents. If the source of uncertainty is purely real, however, we show that currency trading could lead to Pareto improvement.

This paper also provides a framework to examine the question of determinacy of exchange rate. With stochastic monetary transfers, we show that the exchange rate is determinate in the steady state. With real uncertainty, the exchange rate is indeterminate. Thus, whether the exchange rate is determinate or not also depends on the nature of uncertainty.

Although our results are not meant to yield any direct policy recommendations at this stage, they do call for a more careful and extensive study on currency convertibility and exchange rate indeterminacy issues using incomplete contingent market frameworks.

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