World Economy Modeling : What is the mechanism of the world economy?

By

OH, Seungtae

THESIS

Submitted to

KDI School of Public Policy and Management

In Partial Fulfillment of the Requirements

For the Degree of

MASTER OF PUBLIC POLICY

2020

World Economy Modeling : What is the mechanism of the world economy?

By

OH, Seungtae

THESIS

Submitted to

KDI School of Public Policy and Management

In Partial Fulfillment of the Requirements

For the Degree of

MASTER OF PUBLIC POLICY

2020

Professor Tabakis, Chrysostomos

World Economy Modeling : What is the mechanism of the world economy?

By

OH, Seungtae

THESIS

Submitted to

KDI School of Public Policy and Management

In Partial Fulfillment of the Requirements

For the Degree of

MASTER OF PUBLIC POLICY

Committee in charge:

Professor Tabakis, Chrysostomos, Supervisor ______________

Professor Baek, Ji Sun ______________

Approval as of December, 2020

Abstract

This article covers the mathematical modeling of the world economic system. This research activities aim to provide modeling for the simulation of the world economy. They are anchored to the research question; "What is the mechanism of the world economic system?". For the macroscopic approaches, world-system analysis with core countries and peripheral countries, and global value chains are reviewed for the structure examination of the world economy. Business cycles (BCs), economy synchronization and financial contagions are also reviewed as organic behaviors of the world economy. Modeling has been taken by integrating the theories of economic textbooks (micro-economics, macro-economics and international trade) mathematically in a single tone. So, the model can show multiple interactions among markets and participants. General Equilibrium Theory (GET) also deals with the same scope of the integration. However, the models in the thesis have the distinct coverage of transient market changes while GET concerns only equilibrium states. The simulator of the world economy can be built through networking the models of the countries. The analysis of real network patterns of the world has been taken in growth rates of countries, commodity trades, capital flows. Countries in high income group dominate around 70 % of the world economy growth. Top 10 countries in the commodity trade occupy 50 % of the whole international trade. The United States, China and Germany rule the global commodity trade with tight links. The networks of net foreign direct Investment (FDI) are used to examine the capital flows due to the lack of open data of debt networks. The United States and Japan are respectively the biggest investing and invested countries with a tight link while China as a big country in the commodity trade networks is placed as a peripheral country in the net FDI networks. These analyses can provide

the networking directions of the models. To check the weakness of the model, it is examined to apply to the economy crises. In economy crises, the bubble generation with excessive credit creation and its burst with default should be processed. However, the model based on the theories of the economic textbooks does not have the process. The process can be easily added on the model while this article did not complement the model by adding the credit process. This thesis has the economic modeling for the world economy simulator through the mathematical integration and representation of the theories in the economic textbooks. And, the modeling and the research activities reveal the features and mechanism of the world economy.

Contents

1. Introduction 1

2. Physiology of the world economy 4

2.1 World-systems analysis 4

2.2 Global value chains 6

2.3 Business cycles 8

2.4 Economy synchronization 10

2.5 Financial contagion 14

3. Economy Modeling 16

3.1 National economy 17

3.2 Players 22

3.2.1 House holder 22

3.2.2 Firm 27

3.2.3 Government 33

3.2.4 Foreign countries 35

3.3 Markets 39

3.3.1 Market dynamics 39

3.3.2 Commodity market 45

3.3.3 Labor market 54

3.3.4 Financial market 59

3.3.5 Exchange market 62

3.4 Trade 65

4. Economy network pattern 82

4.1 Growth rate pattern 82

4.1.1. Income group and regional contributions 83

4.1.2. Growth rate correlations 87

4.2 Commodity trade pattern 91

4.3 Capital flow pattern 99

5. Economy crises 104

5.1 Great depression 104

5.2 1997 Asian financial crisis 105

5.3 Subprime mortgage crisis 106

5.4 Origin of economic crises and model compatibility 108

6. Summary 111

References 114

Appendix

Chapter 1. Introduction

This article covers the mathematical modeling and networks of the world economic system. The artificial intelligence (AI) has been tried to take the roles of humans in every area [1], and AI has been challenged to find solutions of economic problems for the economic dream; stable and prosperous economic system. Todays, most predictions and policies of the modern economic system are based on the statistical extrapolation of the past similar cases. However, the changes of the world get accelerated everywhere, and uncertainty gets higher, and the knowledge from the previous experiences does not seem to be enough to give the proper solution of the future economic problems. Frankly, we do not have well organized models of the economic system to be provided to AI. Considering contemporary AI opened by Hinton [2] is just an algorithm to solve the given problem, AI does not have free will or ability to rebuild the given ambiguous problem though AI itself is a powerful tool. AI can show incredible performance only under well-defined problems with nice and enough data and delicately designed learning strategies. So, the work in this thesis is building the mathematical models of the world economic system to be provided to AI or a world economic simulator. The research activities eventually address the research question; "What is the mechanism of the world economic system?".

The most previous studies of economics seem to be fragmented into micro-economics, macro-economics, international trades and specific topics, and they are explained mostly by graphical diagrams, and the most previous studies in the economic problem deal with its equilibrium. The real economic system has been working in an integrated way over the world [3]. So, General Equilibrium Theory (GET) dealing with the multiple interactions among markets or participants has been derived in the 1950s. [4 - 6] However, the theory does not seem to be enough to describe the real world economic system since it concerns only equilibriums of markets. The real economic system usually takes a leap before reaching its equilibrium due to the occurrence of its continuous and abrupt condition change caused by the proactive actions of the market participants. Moreover, due to its complexity considering a lot of parameters of the relevant markets, GET does not seem to be widely used in actual economic analyses. The graphical explanation widely used in economics can be intuitive, but it has a limitation to describe thoroughly the working mechanism of the economic system. So, this thesis does deal with the transient and simultaneous behaviors and networks of the economic systems based on the integration and modification of previous theories in economic textbooks. Though the works in this thesis may not cover all the real cases, they will provide a mathematical backbone of the modeling of the world economic system at least, and they can provide the collaboration protocol among economists, physicists, mathematicians and policy makers.

The coverage of this thesis is larger than ordinary research thesis since the research question of this thesis is broad. Thus the thesis doesn’t follow the ordinary structure of the theses with a specific research question or hypothesis. Rather, this thesis is a kind of a journey exploring the breathing mechanism of the world economy. So, the structure of this thesis has the form narrowing scopes from the overall world-systems to individual markets or networks. Chapter 2 has the macroscopic reviews on the world economic system. This chapter shows the physiology of the world economic system. As for its research items, “world-systems theory”, “value chain”, “business cycles”, “economy synchronization”, and “financial contagion” are reviewed. Chapter 3 rearranges the theories of the economy textbooks, and it integrates them in a single tone of mathematical expressions. This chapter is designated to build the mathematical models of national economy. Chapter 4 presents economy networks and patterns from the real data. This chapter proposes the piling or assembling ways of the models in Chapter 3 to simulate the world economy. For checking the limitation of the models, economy crisis will be reviewed, and the application of the model to the economy crises will be discussed in Chapter 5. This thesis will be closed with a summary of Chapter 6.

Chapter 2. Physiology of the World Economy

This chapter will show croquis of structures and movements of the world economy. The overview through related literature reviews is the first procedure to figure out the research question of “What is the current figure of the world economy?”. The research question can be roughly answered by the studies about “world-systems analysis”, “global value chain”, “business cycles”, “economy synchronization”, and “financial contagion”. According to the studies, the economy system need to be treated as a living organism like a creature rather than a machine since the economy system adjusts its structure or shape to fit to its environment. The world-system analysis figures the changing shape of the economy with a head, a body and a tail. The global value chain evolved from global trade studies shows the biological features of its vitality based on the international trade ecology. Business cycles reveal the physiology or bio-rhythms of the economy. Studies of economy synchronization show the co-movements and linkages among economies like an organ, and the processes are explained by financial contagion studies.

2.1 World-systems analysis

World-systems analysis is providing the historical and fundamental scope of the world economy. It does not cover only economic aspect but also political and cultural aspect. The world-systems analysis is started to explain the reasons for the rise and fall of nations of the world history and social changes. In this analysis, the world is divided into core, semi-periphery, or periphery countries using the transnational labor productivity. [7 - 12] So, core countries have higher skill, capital-intensive production while the others have low-skill, labor-intensive production and extraction of raw materials. The core countries monopolized the capital-intensive production. Due to the monopoly of the core countries, the semi-periphery and periphery countries could provide only workforce and raw resources. These characteristics were regarded as the main reason that the core countries dominate the world economics. However, due to some reasons including the transport revolutions, the world hegemon passed from the Netherlands to the United Kingdom and again from the United Kingdom to the United States.

World-systems analysis has been developed by Immanuel Wallerstein in the 1970s, and the world system was characterized as a set of mechanisms which redistributes surplus value from the peripheral countries and the core countries. [10] According to Wallerstein (1974), the control power of the world economy is given to the West as the core countries by unequal development caused by the industrialization and capitalist economy in his historical reviews. [10] Today's world-systems are originated to English industry revolution in the 16th century, and today's globalization was referred to be the result of the capitalist development over the past 500 years. Janet Abu Lughod [13], Andre Gunder Frank [14] and Andrey Korotayev [15] have researched the historic world-systems, and the world-systems in the 11th century are graphically shown in Figure 1. In short words, the world-systems are reflected on the trade patterns, in which the core countries occupy a dominant position with higher productivity than the periphery countries. Today's trade pattern is based on global value chains.

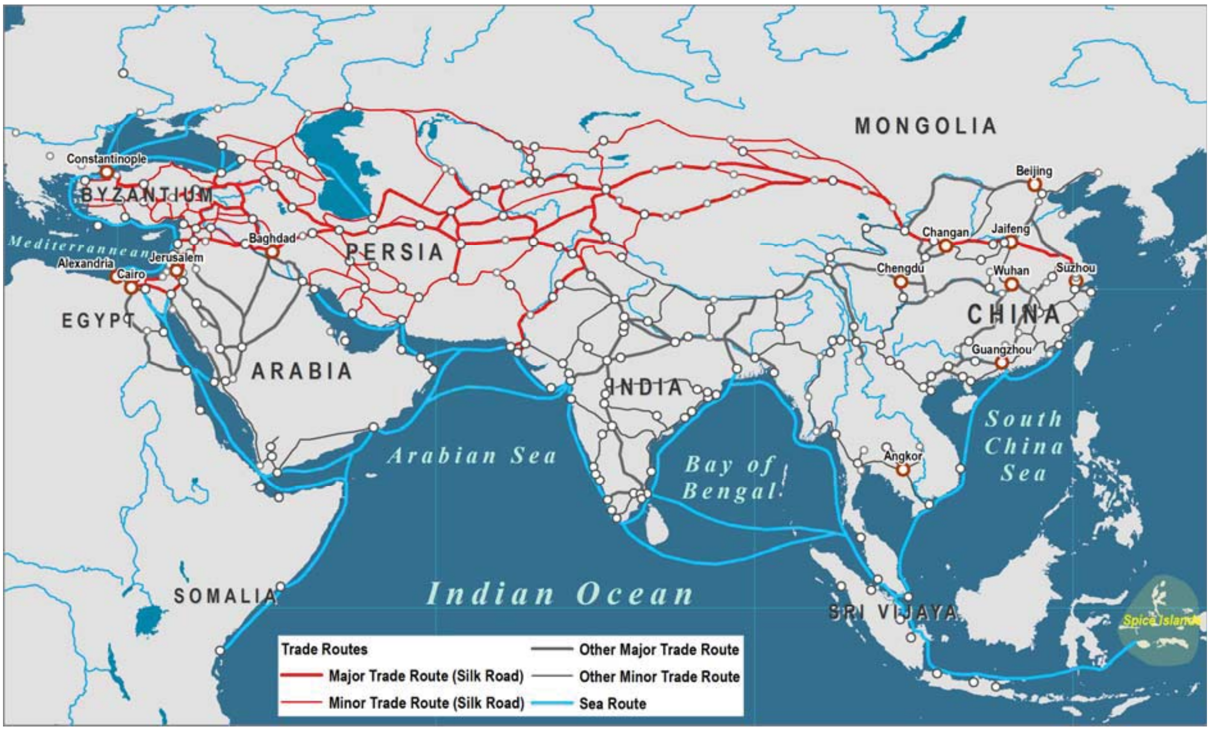


Figure 1. World systems in the 11th and the 12th centuries

2.2 Global Value Chains

In development studies, the global value chain (GVC) describes the all cross-geographic activities in designing, materials, marketing, distributions and customer services. [16] It is similar to industrial value chain but encompasses the cross-national activities. It is a sort of the modern trade pattern with fragmented production process. So, it is used in the analysis of international trade. The Ricardian trade model [17] is regarded to be suitable to explain the GVC. [18]

In Reference 16, two types of governance are introduced as buyer-driven chains and producer-driven chains. In non-durable consumer products, final buyers such as retail firms govern the value chains, and it is the buyer-driven chains. [16] On the contrary, in technology intensive chains, the technological competence determines the lead firms, and it is the producer-driven chains. [16] Current researches of GVCs identify 5 chain patterns (hierarchical chains for vertically integrated firms, captive chains having intermediate customers, relational chain with mutual dependence, modular chain with suppliers' independent competence, and market chain with little interaction beyond exchanging). [19] Only hierarchy chain is referred to the producer-driven governance, and the rest 4 chain patterns are presented in the buyer-driven governance. [19] The factors to determine the type of the chain are referred to "complexity of transactions", "codifiability of transactions" and "competence of suppliers". [19] The 5 patterns seem to be just 5 categories depending on the chaos degrees of the networks since the governance presents the network nature: the governance of the producer gets weaker as the chaos degree of the network increases. From 1980s, outsourcing and subcontracting of multi-national enterprises have started, and these trends were rapidly accelerated by information and communication technologies (ICT) [21]. Foreign direct investment (FDI) is also referred to be the major driver to expand the growth of GVCs. [ref-gcv7] Globalization studies identified four dimensions of GVC shaping: (a) an input-output structure: the process transforming raw materials to final products, (b) a geographical consideration, (c) a governance structure of the chain, and (d) an institutional context. [23] For an example, the value chain of global agriculture is illustrated in Figure 2 with outcome types and geographical positions. Over the last 30 years, the cross-national outsourcings and alliances have been less vertically integrated and more free marketed. [19]

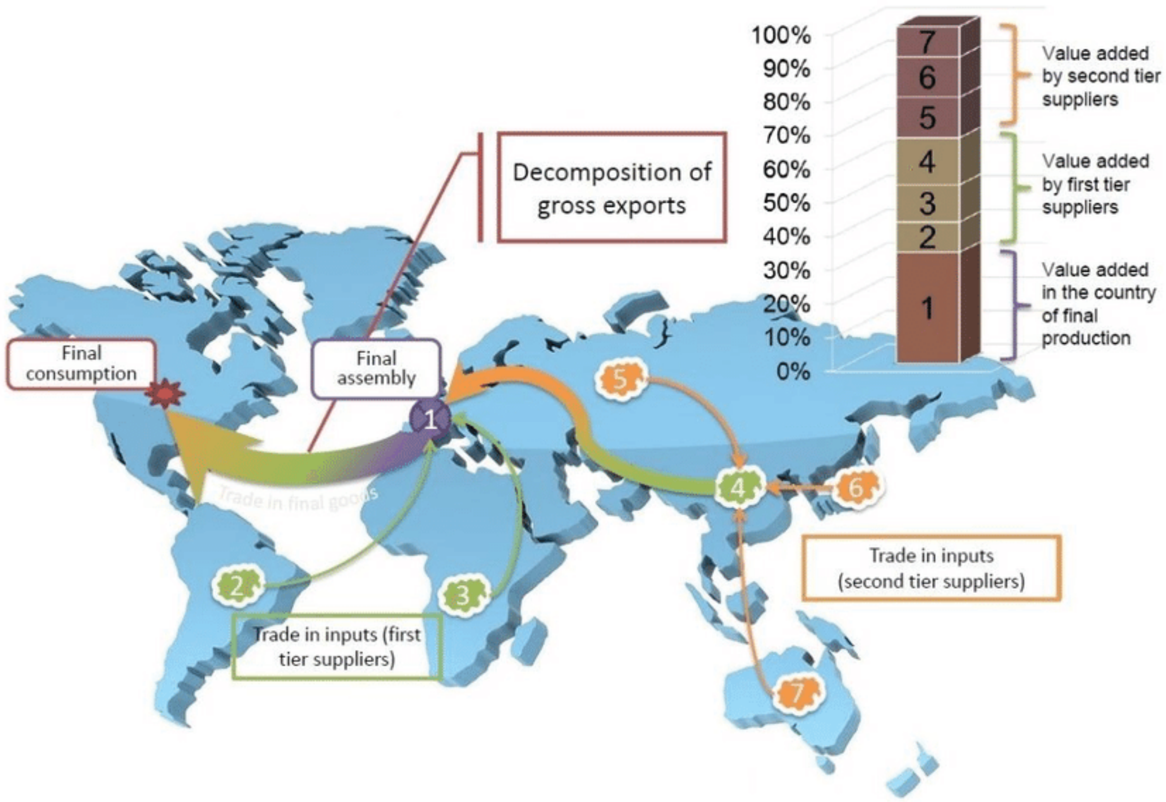


Figure 2. Global agriculture value chain [24]

2.3 Business cycles

The business cycle is the oscillation (booms and recessions) of the gross domestic product (GDP) or the gross national product (GNP) in a long term as shown in Figure 3, and it is referred to the trade or economy cycle. These fluctuations induced by endogenous causes are not exactly predictable since they do not have uniform periodicity or fixed pattern. However, it is considered to have 4 stages: expansion (low interest rates, increase in production and prices), crisis (firms' bankruptcies and crash of stock price), recession (low interest rates, drops in prices and outputs), and recovery (stock price recovery due to the decreased prices and incomes). [25] The business cycles are categorized by their periodicity into four cycles: Kitchin inventory cycle of 3 to 5 years, Juglar fixed-investment cycle of 7 to 11 years, Kuznets infrastructural investment cycle of 15 to 25 years, and Kondratiev wave or long technological cycle of 35 to 60 years. [26] Recent researches using spectral analysis identified that Kondratiev and Kuznets waves present in the world GDP dynamics statistically. [27] For the causes of the business cycles, several explanations were proposed. Neo-classical economists argued exogenous causes such as World War II while Keynesians argued endogenous causes derived by under-consumptions. [28, 29] In mainstream economics, the business cycles are seemed to be essential since it is caused by the summation of random causes. [30] In the Keynesian, cycles in output by the distribution of income between business profits and workers' wages were high-lightened for the causes. The explanations form Marxian economics are impressive to point out the market economy paradox: profit as the main engine of the market economy forces capital profitability to induce massive unemployment and succeeding recurrent business failure. The remaining capital is finally centralized and profitability is recovered. In the long run, these crises will be severe and the market economy will fail at the end. [31] For alternative explanations, political business cycle [32] and yield curve [33] were considered. In the point of view from the author as a physicist, every system has its own resonance frequency, and the system will get mal-functioned at around the resonance frequency. And the system has its own restriction functions to avoid the system-breaking phenomenon intrinsically.

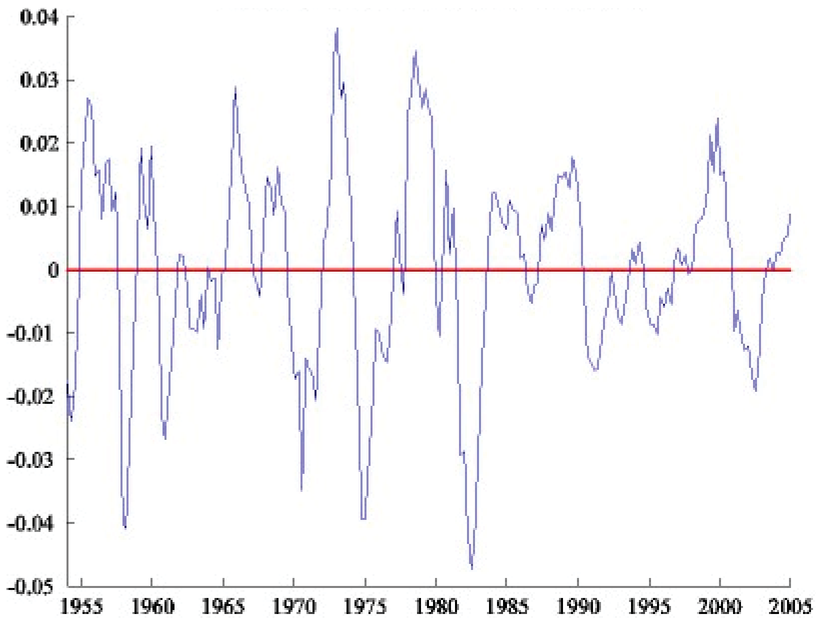


Figure 3. Deviations from the long-term United States growth trend (log GNP), 1954–2005 [34]

2.4. Economy synchronization

When a system has a channel linked to external forces, the system would be influenced by the external force. The closer the channel is to the core factors of the system, the more severe its impact is. The linkage effects of economies appear as business cycle synchronizations. So, economy synchronizations seem to be regarded as natural phenomena, and the synchronized movements of economic growth rates are referred to be caused mainly by two reasons:

1. Trade and capital flows due to the globalization [35 - 37]

and

2. The same currency group due to the same monetary policy adaptation [38 - 40].

Considering that the channels of a country to an external force are its goods market and finance market, the reasons are quite reasonable.

One of the most focused research area in the synchronization of the business cycles is synchronicity assessments. Direct correlation measurements do not always reveal the business cycle synchronization of economies due to the stochastic nature of economy. [41] Nevertheless, simple measures such as maximal windowed correlations have been used with long term datasets, and there are still new techniques being researched for the synchronicity assessments. [41] The studies are also taken in mathematical fields. Xi and colleagues have proposed entropy model in the synchronization analysis of the business cycles as shown in Figure 4. [42] Multivariate singular spectrum analysis (M-SSA) was also applied to identify shared oscillatory modes of the business cycle fluctuations and to analyze structural changes in the cluster configuration of synchronization. [43] The M-SSA studies have shown that many economies shared a common oscillation mode of business cycle activity, and the primary oscillation mode had 7-11-year period. Several major events in the world economy of the post war era were linked to these lower-variance modes. [43, 44]

For the feature extractions of the business cycle synchronization, Crowley and Trombley (2015) showed that monetary unions were greatly synchronized in inflation and unemployment using recurrence plot approach, and that each monetary union had its own internal dynamics in GDP growth rates. [41] Oliveira and Araujo (2009) high-lightened the great collapse of the trade flows during global finance crisis in 2009 as shown in Figure 5, and the collapse is referred to be the evidence of the great global synchronization. [45] Belke, Domnick and Gros (2017) showed that core countries (Germany, France, Austria, Finland and the Netherlands) in Europe monetary union (EMU) were strongly synchronized, whereas the synchronization of peripheral countries (Portugal, Italy, Ireland, Greece and Spain) was decreased. [46]. This divergence was found to be obvious after the financial crisis in 2009, and they pointed out the problems of the common monetary policy in EMU under the divergence. [46] Most related studies are focusing on the synchronization degrees or features not on the mechanism of the synchronization since it is hard to be figured out by both theoretical model and empirical work. One of the trials to explain the mechanism is financial contagion.

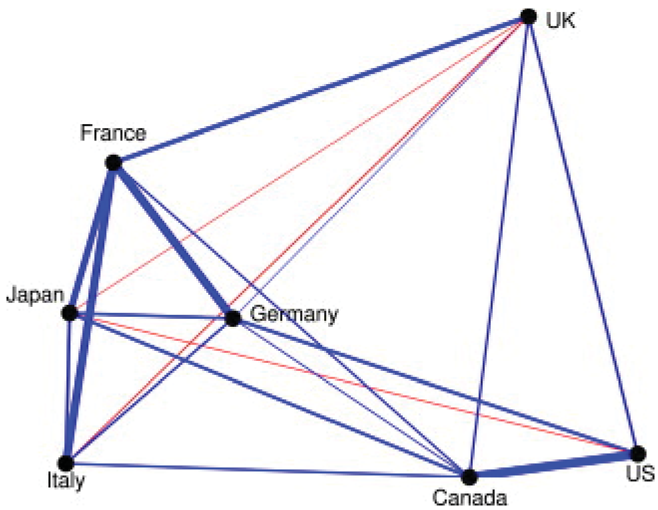


Figure 4. G7 synchronization networks through pairwise entropy test. The blue and red lines are respectively positive and negative interactions, and the thickness of the lines presents the strengths of pairwise interactions. [42]

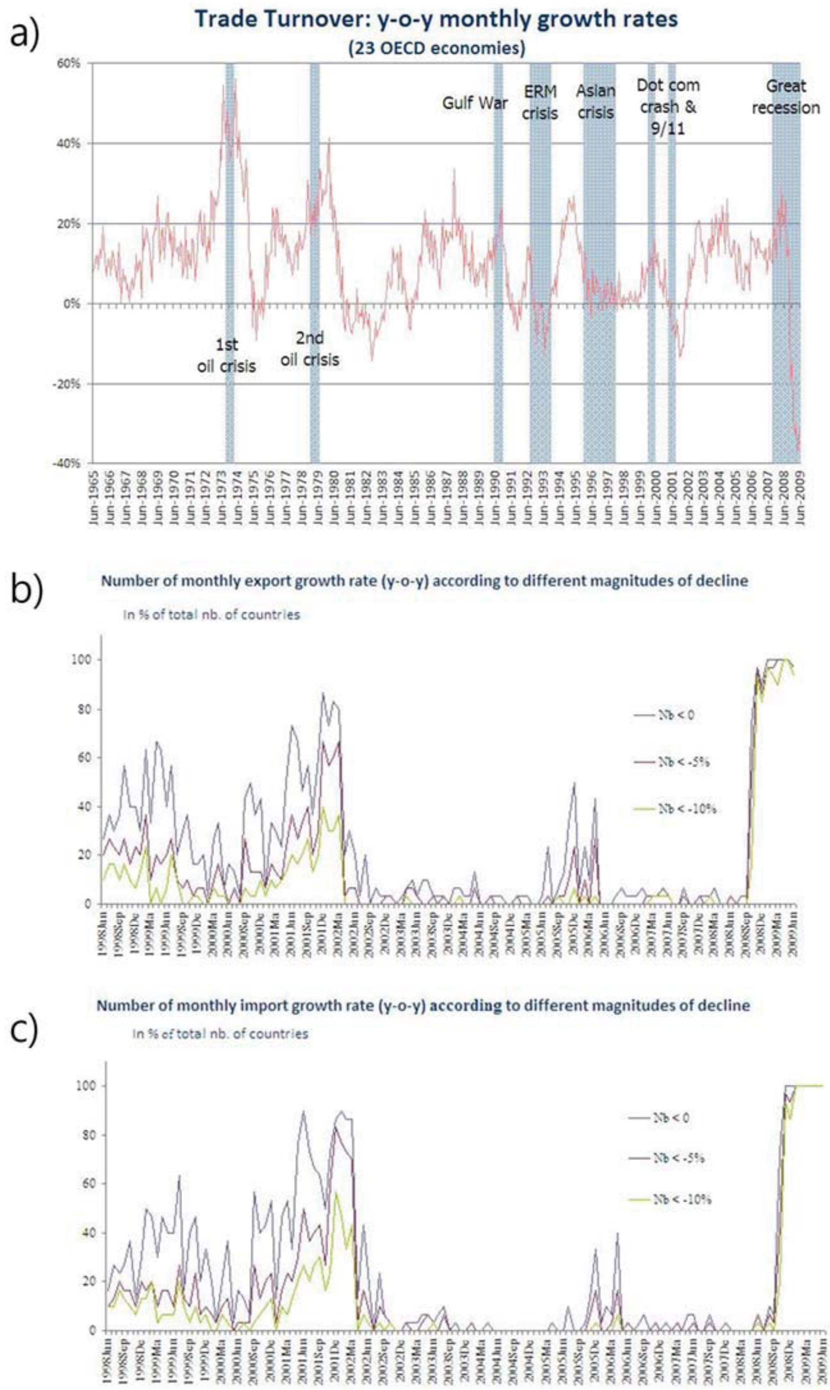


Figure 5. a) Global trade growth rate, b) Percent of countries with negative export growth, c) Percent of countries with negative import growth. [45]

2.5 Financial contagion

Financial contagion is the transmission financial crisis across other financial markets through direct or indirect ways, and it happens in both advanced countries and developing countries. [47] That is regarded as the spillover effect and the financial crisis caused by governments, financial institutions, investors, and borrowers. [48, 49] The fundamental causes are examined through macroeconomic shocks in international or local scales induced by trade links, competitive devaluations, and financial links. [48] A financial crisis in a country is directly transmitted to the rest of the world through foreign direct investment, abroad capital flow, and trade credit reductions since countries are more economically integrated with global financial markets. [48] As for the structural causes to explain the financial contagion, a small liquidity preference and financial fragility were referred to be the keys of the contagion by Allen et al. [50] and Lagunoff et al. [51]. Each group showed the linkages among financial intermediaries respectively through a general equilibrium model [50] and a dynamic stochastic game-theoretic model [51].

Trade shocks are the results of the local impact integration. A sharp depreciation of a country in its crisis will decline asset prices and large capital outflows, and the exports to the country in its crisis will be deteriorated in its trade credit. [49] Competitive devaluation is a currency war in a word. So, every country competes to gain low exchange rate for its currency due to its competitive advantage. Devaluation of a country in a crisis put pressure to lower the currency value of other countries since the devaluation of the country make the countries lose their export competitiveness. [49] Under the circumstance, fear will overwhelm the finance markets since the credit holding countries will sell their holdings of securities, reduce their lending or refuse to postpone loan deadlines. [49] The fear can be induced without any crisis by risk aversion or lack of confidence of the four agents, and the significant player among the four agents is regarded to be the investor. [52] For the econometric models of the contagion, a vector auto-regression used as the standard model can be

$$\begin{pmatrix} p_1(t) \\ p_2(t) \end{pmatrix} = \begin{pmatrix} c_{1,0} \\ c_{2,0} \end{pmatrix} + \begin{pmatrix} c_{1,1} & c_{1,2} \\ c_{2,1} & c_{2,2} \end{pmatrix} \begin{pmatrix} p_1(t-1) \\ p_2(t-1) \end{pmatrix} + \begin{pmatrix} e_1(t) \\ e_2(t) \end{pmatrix} \tag{2.1}$$

where $p_i(t)$, $c_{i,j}$ and $e_i(t)$ are respectively the price of asset i, the element of a contagion matrix and its regression error at time t. [53] This model is quite similar to Markov chains [54]. However, Markov chain could not been found in the contagion studies though Markov chain model is very flexible for inter-correlated circumstances.

Chapter 3. Economy Modeling

This chapter will show you the mathematical integration of the theories in economic textbooks. This work is to reveal the mechanism of economy for the research question: “How does the world economy work?”. Micro-economics, macro-economics and international trade are regarded as the back-bone of the economics. Though they describe the structure of the economy and the actions of the players, they are rarely used as analysis or explanatory tools in the actual researches. I think that is caused by knowledge fragmentation since each model seems to work independently although every economy is simultaneously linked. Moreover, the models are not good to reflect the real cases since they are mainly focusing on their equilibrium states although the situations of the economy are continuously changing before they reach their equilibriums. And the transient changes of the economic system were rarely concerned. Moreover, in modern economies, foreign economies in outside of a country are getting more influencing on the economy of a country due to globalizations, but the economy is treated as a static object not a living organism in the text books. Though I cannot resolve all the problems by myself at once, the integration of the models in a single mathematical tone would be quite meaningful for its application of the model after its upgrade or reflections of other factors. For the first reflections of the transient behaviors of the market, here the model modification can be also found to present the dynamics of the economy. The features of the previous chapter are tried to be understood on the previous theories, and the limitation of the theories will be commented.

The theories and models in this chapter are based on the books of the microeconomics [55, 56], macroeconomics [57, 58] and international trade [59, 60]. In Section 3.1, the overall structure of the national economy will be described in Section 3.1, and the actions of the players will be described from in Section 3.2. As for the market actions, the dynamics of a market nature and four markets will be discussed in Section 3.3, and international trade will be described in Sections 3.4.

3.1 National economy

National Economy can be structured with 4 markets (commodity market, labor market, finance market and exchange market) and 4 players (house holder, firm, government and foreign actor) as shown in Figure 6. In conventional macroeconomics dealing with national economic system, only 3 markets (commodity market, labor market and finance market) and 3 players (house holder, firm and government) mainly are concerned. However, globalization makes external economies non-negligible in a national economy. Thus, the conventional national economic model with 3 markets and 3 players should be modified to the model with 4 markets and 4 players as shown in Figure 6. There are many individual participants in a player category or in a market category, but they are assumed to be identical, and they in a category will be treated as a single one.

All players move to maximize their own utility functions, and they have their own working markets. The house holder moves to maximize his happiness with his utility function by consuming commodities or saving money under his preference. So, he provides his labor to get the wage as his income from the firm through the labor market. After paying taxes to the government, he will consume some commodities and pay some of the net income at a given price of the commodity market, and the residues will be provided as savings in the finance market. His allocations of the net income are based on his preference, and he can give up his participation in the finance markets due to his speculative motivation when the interest rate of the finance market is lower than his limit interest rate.

The firm produces and supplies commodities to the commodity market to maximize the profit of the firm under the given market price. For the production, loans from the finance market, labors from the labor market and facilities from the commodity market are used. For the returns of the labor usages, the firm should pay the house holder a wage through the labor market, and the cost of the facilities will be paid as their investments in the commodity market. So, the firm will allocate the loan from the finance market with their own production functions of labors and investments. The firm will get the income as the return of the supplied commodities, and the income of the firm is the national total yield or gross domestic production (GDP). From the income, taxes and the interest as the loan cost are paid respectively to the government and the finance market.

The government moves independently to moderate the markets for sound growth. That means that the government is hardly constrained. The government collects taxes from the house holder and the firm, and it will consume goods in the commodity market as its fiscal expenditure, and it can determine the money flow with selling or buying treasury bonds or with controlling BIS ratio or base interest rate. The foreign actor moves when there is a price or interest rate difference between the foreign market and the domestic market. The foreign actor can access the commodity market and the finance market through the exchange market.

The markets determine their trade exchange rates (prices) by matching demands and supplies. In the labor market, the participants are the house holder and the firm, and they trade labor at a wage. In the commodity markets, the firm is a major supplier, and the house holder and the government would be the main consumer, and the foreign actor will be either a supplier or a consumer with the price level difference. They trade commodity and the commodity price will be determined by matching the total supply and the total consumption. In the finance market, the major consumer is the firm, and the house holder and government are the suppliers. The foreign actor will be either consumer or supplier by difference in its interest rate. The finance market determines the interest rate by matching money supplies and demands. The exchange market is the gate of the foreign actor. The exchange market determines the currency exchange rate by matching foreign currency supplies (commodity exports or foreign savings in finance market) and foreign currency demands (commodity imports or deposits in foreign finance market). The relations between the 4 players and the 4 markets are presented in Figure 6. In the model with 4 players and the 4 markets, the conventional equation of total yield (Y) is also valid as

$$Y = C + G + I + NX \qquad (3.1)$$

where C, G, I and NX are respectively the house holder's consumption, the government expenditure, the firm's investment and net exports to foreign countries. Eq. (3.1) was checked by matching the inputs (C, G, I and NX) and the output (Y) of the commodity market. So, GDP means the market value of the commodity market. The inputs and outputs of the firm give

$$Y + loan = w + T_F + I + S_F \qquad (3.2)$$

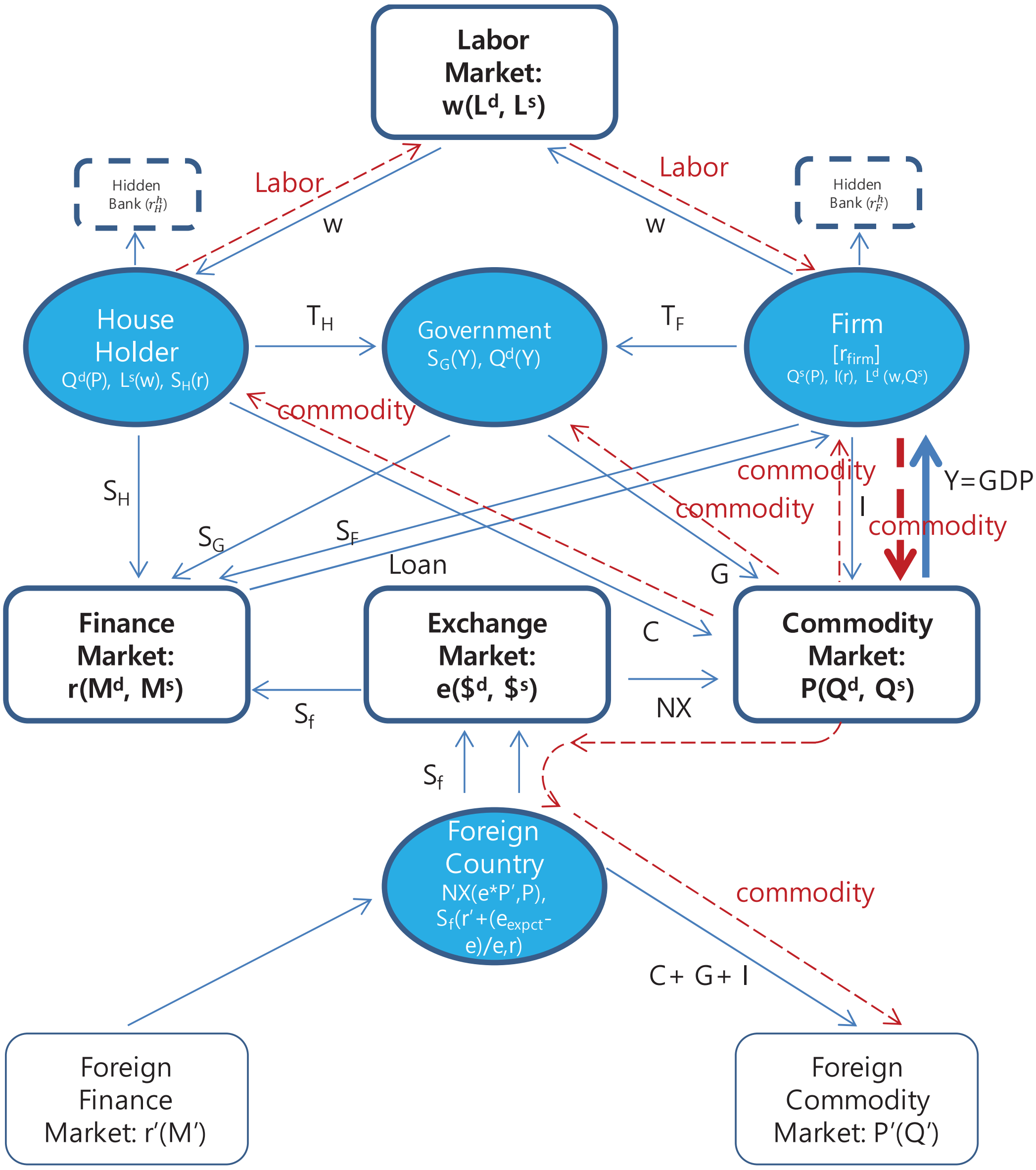
where $w = T_H + C + S_H$ and $loan = S_H + S_G + S_F + S_f$, and Eq. (3.2) can be rewritten as

$$Y = C + I + G - S_f \qquad (3.3)$$

since $G + S_G = T_H + T_F$. From Eq. (3.1) and Eq. (3.3),

$$NX + S_f = 0 \tag{3.4}$$

can be derived. Eq. (3.4) means that the net exports and foreign saving mutually suppress to reach their adequate levels. Therefore, when a country set its financial policy to increase the foreign saving, then its export will be declined.



T: tax	I: invest	Blue solid line : money flow
w: wage	NX: net export (export-import)	Red dashed line : item flow
S: saving	Y: yield (GDP)	Circle : actor
L: labor	Q: quantity	Square : market
r: interest rate	M: money quantity	
r_{firm}: profit rate	e: exchange rate	
C: consumption	$: key currency	
G: government expenditure		

Figure 6. The structure of a national economy.

3.2 Players

3.2.1 House holder

House holders take roles of consumers in the economy. Let us assume that all consumers are identical, and there is a single kind of commodity. So, the consumers will be treated as a single consumer. He spends money in purchasing the commodity and the residue of their incomes will be saved to maximize his utility function. The utility function (U) based on Cobb-Douglas model [61] can be expressed as

$$U = {S_H}^{\alpha_c + \gamma_c r} Q_c^{\beta_c} \tag{3.5}$$

where Q_C, S_C and r are respectively his consumption quantity of the commodity, Saving quantity and its interest rate. α_c and β_c in Eq. (3.5) represent respectively the saving preference with zero interest rate and the commodity preference of the consumer, and γ_c is the scale coefficient of the preference of the interest rate. Now his income will be expressed as

$$P_c\, Q_c + S_H - (W - T_{consumer} + Loan_H) = 0 \tag{3.6}$$

Where P_c , Q_c, S_C, W, $T_{consumer}$ and $Loan_H$ are respectively the price of the commodity, its consumption quantity, saving quantity, income, consumer tax and consumer's loan from financial market. Using Lagrangian multiplier,

$$L = U - \lambda(budget\ contraint), \tag{3.7}$$

$$L = {S_H}^{\alpha_c + \gamma_c r} Q_c^{\beta_c} - \lambda(P_c\, Q_c + S_C - W_{net}), \tag{3.8}$$

$$\frac{\partial L}{\partial S} = (\alpha_c + \gamma_c\, r){S_H}^{\alpha_c + \gamma_c r - 1} Q_c^{\beta_c} - \lambda = 0, \tag{3.9}$$

$$\frac{\partial L}{\partial Q_c} = \beta_c Q_c^{\beta_c - 1} {S_H}^{\alpha_c + \gamma_c r} - \lambda P_c = 0 \,, \tag{3.10}$$

and

$$\frac{\partial L}{\partial \lambda} = P_c Q_c + S_H - W_{net} = 0 \tag{3.11}$$

can be derived where $W_{net} \equiv W - T_{consumer} + Loan_C$, and using Eq. (3.9) and Eq. (3.10),

$$(\alpha_c + \gamma_c r) P_c Q_c = \beta_c S_H \tag{3.12}$$

can be given. As for consumption quantity and saving quantity,

$$Q_c = \frac{\beta_c}{(\alpha_c + \beta_c + \gamma_c r) P_c} W_{net} \tag{3.13}$$

and

$$S_H = \frac{\alpha_c + \gamma_c r}{\alpha_c + \beta_c + \gamma_c r} W_{net} \tag{3.14}$$

can be determined with Eq. (3.11) and Eq. (3.12). The consumption quantity of Eq. (3.13) with $\alpha = \beta = \gamma = 1$, $W_{net} = 10$ and $r = 0.05$, and the saving quantity of Eq. (3.14) with $\alpha = \beta = \gamma = 1$ and $W_{net} = 10$ were plotted respectively in Figure 7 a) and b).

Using Taylor expansion [62], Eq. (3.13)will be simplified to

$$Q_c = \frac{\beta_c W_{net}}{(\alpha_c + \beta_c + \gamma_c r) P_0 (1 + \frac{(P_c - P_0)}{P_0})} \tag{3.15}$$

$$\approx \frac{\beta_c W_{net}}{(\alpha_c + \beta_c + \gamma_c r) P_0} \left(1 - \frac{(P_c - P_0)}{P_0} \right) \tag{3.16}$$

$$= -\frac{\beta_c W_{net}}{(\alpha_c + \beta_c + \gamma_c r){P_0}^2}(P_c - P_0) + \frac{\beta_c W_{net}}{(\alpha_c + \beta_c + \gamma_c r)P_0} \tag{3.17}$$

since $\frac{1}{1+x} \approx 1 - x$ for $|x| \ll 1$. So, the consumption quantity can be finally expressed in a linear equation form as Eq. (3.17). The consumption quantity is the commodity demand from the consumer. So, Eq. (3.17) can be the demand curve as

$$D(P) = -\alpha_D(P - P_0) + Q_0 \tag{3.18}$$

around P_0. The coefficients $(\alpha_D,\ Q_0)$ are

$$\alpha_D = \frac{\beta_c W_{net}}{(\alpha_c + \beta_c + \gamma_c r){P_0}^2} \tag{3.19}$$

and

$$Q_0 = \frac{\beta_c W_{net}}{(\alpha_c + \beta_c + \gamma_c r)P_0} \tag{3.20}$$

from Eq. (3.17). Eq. (3.19) presents the nature of the demand curve. α_D represents the demand elasticity against price changes. And α_D is proportional to its market value (Y) or size $(P_0 Q_0)$ since the consumer's income (W) is proportional to the market value (Y). A product with higher price will be robust against its price changes than a product with lower price, when P_0 is the market equilibrium price. The demand curve of Eq. (3.18) presents the most rational action of the consumer. However, the consumer acts with his own prediction or he give up more participation in the market. Thus, Eq. (3.18) should be modified with these factors into

$$D(P) = \begin{cases} -\alpha_D(P - P_0) + Q_0 + \alpha_D^{\Delta p}\dfrac{dP}{dt} + \alpha_D^{\Delta S}\dfrac{dS}{dt} & when\ P < p_{max} \\ D_{min} & when\ P \geq p_{max} \end{cases} \tag{3.21}$$

where $D_{min} = -\alpha_D(p_{max} - P_0) + Q_0$. $\alpha_D^{\Delta p}$ and $\alpha_D^{\Delta S}$ are consumer's prediction factors respectively for price changes and supply changes, and p_{max} is the maximum price for consumer's market participation.

The saving quantity of Eq. (3.14) can be also simplified with Taylor expansion as

$$S_H = \frac{\alpha_c + \gamma_c\, r}{\alpha_c + \beta_c + \gamma_c\, r} W_{net} = \frac{\alpha_c + \gamma_c\, r}{(\alpha_c + \beta_c)\left(1 + \frac{\gamma_c\, r}{\alpha + \beta}\right)} W_{net} \tag{3.22}$$

$$\approx \frac{W_{net}}{\alpha_c + \beta_c}(\alpha_c + \gamma_c\, r)\left(1 - \frac{\gamma_c\, r}{\alpha_c + \beta_c}\right) \tag{3.23}$$

$$= \frac{W_{net}}{(\alpha_c + \beta_c)^2}(\alpha_c(\alpha_c + \beta_c) + \beta_c\ \gamma_c\, r - (\gamma_c\, r)^2) \tag{3.24}$$

With ignoring the high order term (r^2), Eq. (4-c-20) can be

$$S_H \approx \frac{\beta_c\, \gamma_c\, W_{net}}{(\alpha_c + \beta_c)^2}\,(r - r_0) + \beta_c\ \gamma_c\, W_{net}\left(\frac{\alpha_c}{(\alpha_c + \beta_c)} + \frac{r_0}{(\alpha_c + \beta_c)^2}\right) \tag{3.25}$$

since r << 1.

From Eq. (3.25), the saving quantity can be expressed in a linear form as

$$S_H(r) = \alpha_{S_H}(r - r_0) + S_{H0} \tag{3.26}$$

where $\alpha_{S_H} = \frac{\beta_c\ \gamma_c\, W_{net}}{(\alpha_c+\beta_c)^2}$, $S_{H0} = \beta_c\ \gamma_c\, W_{net}\left(\frac{\alpha_c}{(\alpha_c+\beta_c)} + \frac{r_0}{(\alpha_c+\beta_c)^2}\right)$ and r_0 is the market equilibrium interest rate. With the additional actions of the consumer's, Eq. (3.26) should be

$$S_H(r) = \begin{cases} \alpha_{S_H}(r - r_0) + S_{H0} + \alpha_{S_H}^{\Delta r}\dfrac{dr}{dt} & when\ r > r_{min} \\ S_{min} & when\ r \le r_{min} \end{cases} \tag{3.27}$$

where $S_{min} = \alpha_{S_H}(r_{min} - r_0) + S_{H0}$. And r_{min} is the minimum interest rate for consumer's market participation. $\alpha_{S_H}^{\Delta r}$ is the contribution coefficient of the interest change rate to the saving.

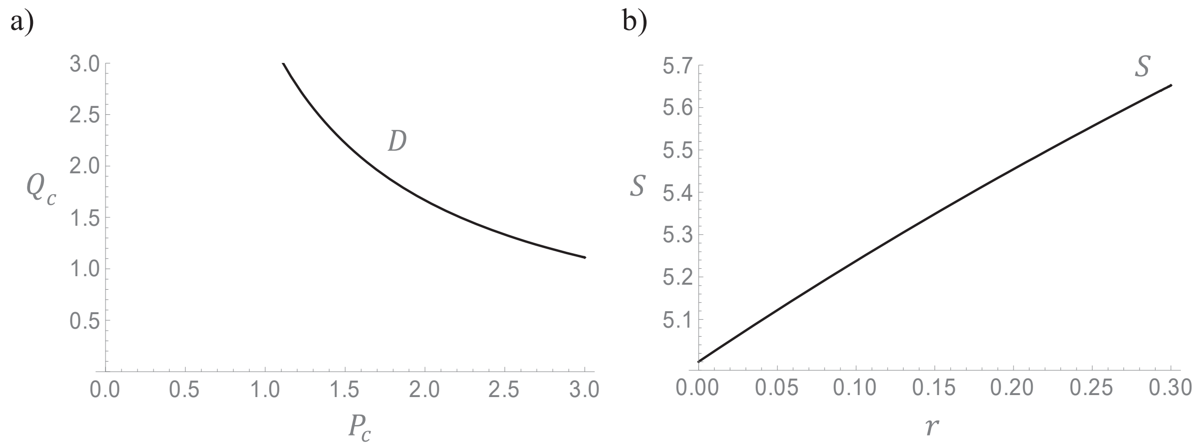


Figure 7. Consumer's actions: a) consumption quantity changes with the commodity price and b) saving quantity changes with the interest rate.

3.2.2 Firm

Firms play roles as producers in commodity markets. Let us assume the firms are identical. So, the firms will be treated as a single firm.

Price driven production

When the market price of the commodity is given, the firm will produce the quantity (Q) of the commodity to maximize its profit under the condition that

$$MC(Q) = P \tag{3.28}$$

where MC, P and Q are respectively the marginal cost, market price and the production quantity of the firm. Let us assume that the firm has the production function as

$$Q = K^{\alpha_{pr}} L^{\beta_{pr}} \tag{3.29}$$

where K and L are respectively capital and labor for the production. α_{pr} and β_{pr} are the coefficients of the production substitutions. The total cost (TC) for the production quantity can be

$$TC = rK + wL \tag{3.30}$$

where r, K, w and L are interest rate, capital quantity, wage and labor quantity. From Eq. (3.29), Lagrangian can be

$$Lagrangian = Q - \lambda(bugent\ constraint) \tag{3.31}$$

for Lagrangian multiplier. Then the Lagrangian multiplier can give

$$\frac{\partial Lagrangian}{\partial K} = \alpha_{pr} K^{\alpha_{pr}-1} L^{\beta_{pr}} - \lambda\, r = 0 \tag{3.32}$$

$$\frac{\partial Lagrangian}{\partial L} = \beta_{pr} K^{\alpha_{pr}} L^{\beta_{pr-1}} - \lambda\, w = 0 \tag{3.33}$$

$$\frac{\partial Lagrangian}{\partial \lambda} = rK + wL - TC = 0. \tag{3.34}$$

The relation between the labor quantity and the capital quantity can be

$$\frac{wL}{rK} = \frac{\beta_{pr}}{\alpha_{pr}} \tag{3.35}$$

From Eq. (3.34) and Eq. (3.35), the required capital quantity and labor quantity can be derived respectively as

$$K = \frac{\alpha_{pr}}{r(\alpha_{pr} + \beta_{pr})} TC \tag{3.36}$$

, and

$$L = \frac{\beta_{pr}}{w(\alpha_{pr} + \beta_{pr})} TC. \tag{3.37}$$

Now, marginal cost should be known since the production quantity is determined by the marginal cost function (MC) in Eq. (3.28) and since the factor quantities (K and L) are determined by the total cost (TC).

From the definition of marginal cost and Eq. (3.30), the marginal cost can be

$$MC(Q) = \frac{dTC}{dQ} = r\frac{dK}{dQ} + w\frac{dL}{dQ}. \tag{3.38}$$

The derivatives of Eq. (3.38) can be derived from Eq. (3.29) as

$$\frac{dK}{dQ} = \frac{1}{\frac{dQ}{dK}} = \frac{1}{\alpha_{pr} K^{\alpha_{pr}-1} L^{\beta_{pr}}} = \frac{K}{\alpha_{pr} Q} \tag{3.39}$$

and

$$\frac{dL}{dQ} = \frac{1}{\frac{dQ}{dL}} = \frac{1}{\beta_{pr} K^{\alpha_{pr}} L^{\beta_{pr}-1}} = \frac{L}{\beta_{pr} Q}. \tag{3.40}$$

So, the marginal cost function of Eq. (3.38) can be

$$MC(Q) = \left(\frac{rK}{\alpha_{pr}} + \frac{wL}{\beta_{pr}}\right)\frac{1}{Q} = \left(\frac{r}{\alpha_{pr}}\frac{\alpha_{pr}}{r(\alpha_{pr} + \beta_{pr})}TC + \frac{w}{\beta_{pr}}\frac{\beta_{pr}}{w(\alpha_{pr} + \beta_{pr})}TC\right)\frac{1}{Q} \tag{3.41}$$

$$= \frac{2}{(\alpha_{pr} + \beta_{pr})} TC \frac{1}{Q}. \tag{3.42}$$

Eq. (3.42) can be rewritten as

$$\frac{dTC}{dQ} = \frac{2}{(\alpha_{pr} + \beta_{pr})} TC \frac{1}{Q}, \tag{3.43}$$

$$\frac{dTC}{TC} = \frac{2}{(\alpha_{pr} + \beta_{pr})} \frac{dQ}{Q}, \tag{3.44}$$

$$\int \frac{dTC}{TC} = \frac{2}{(\alpha_{pr} + \beta_{pr})} \int \frac{dQ}{Q}, \tag{3.45}$$

and

$$\ln TC = \frac{2}{(\alpha_{pr} + \beta_{pr})} \ln Q + c_{scale1}. \tag{3.46}$$

The total cost (TC) is

$$TC = c_{scale2} Q^{\frac{1}{(\alpha_{pr} + \beta_{pr})/2}}, \tag{3.47}$$

and the marginal cost (MC) is derived as

$$MC = \frac{dTC}{dQ} = \frac{c_{scale2}}{\left(\alpha_{pr} + \beta_{pr}\right)/2} Q^{\frac{1}{(\alpha_{pr}+\beta_{pr})/2}-1}, \tag{3.48}$$

$$MC = \frac{c_{scale2}}{(\alpha_{pr} + \beta_{pr})/2} \left((Q - Q_0) + Q_0\right)^{\frac{1}{(\alpha_{pr}+\beta_{pr})/2}-1}. \tag{3.49}$$

Using Taylor series:

$$(a + x)^n = a^n + n\, a^{n-1} x + \frac{n(n-1)}{2}\, a^{n-2} x^2 + \cdots, \tag{3.50}$$

Eq. (3.48) can be a series form of

$$MC = \frac{2\, c_{scale2}}{(\alpha_{pr} + \beta_{pr})} \left(Q_0^{\frac{1}{(\alpha_{pr}+\beta_{pr})/2}-1} + \left(\frac{2}{(\alpha_{pr} + \beta_{pr})} - 1 \right) Q_0^{\frac{1}{(\alpha_{pr}+\beta_{pr})/2}-2} (Q - Q_0) + \left(\frac{2}{(\alpha_{pr} + \beta_{pr})} - 1 \right)^2 Q_0^{\frac{1}{(\alpha_{pr}+\beta_{pr})/2}-3} (Q - Q_0)^2 \ldots \right). \tag{3.51}$$

Finally, the marginal cost is derived as

$$MC(Q) \approx \frac{1}{\beta_S}(Q - Q_0) + P_0 \tag{3.52}$$

where $\frac{1}{\beta_S} \equiv \frac{2\, c_{scale2}}{(\alpha_{pr}+\beta_{pr})} \left(\frac{2}{(\alpha_{pr}+\beta_{pr})} - 1 \right) Q_0^{\frac{2}{(\alpha_{pr}+\beta_{pr})}-2}$ and $P_0 \equiv \frac{2\, c_{scale2}}{(\alpha_{pr}+\beta_{pr})} Q_0^{\frac{2}{(\alpha_{pr}+\beta_{pr})}-1}$.

Using Eq. (3.28) and Eq.(3.52) , the supply curve (S) can be derived as

$$S(P) = \beta_S(P - P_0) + Q_0 \tag{3.53}$$

since the supply quantity (S) into the commodity market is the production quantity (Q) of the firm. So, once the price market is given, the production quantity of the firm can be determined by Eq. (3.53).

When the price is given, the total cost can be

$$TC = c_{scale2}(\beta_S(P - P_0) + Q_0)^{\frac{2}{(\alpha_{pr}+\beta_{pr})}} \tag{3.54}$$

using Eq. (3.47) and Eq. (3.53). Using Taylor expansion of Eq. (3.50), Eq. (3.54) around P_0 can be a linear form as

$$TC \approx c_{scale2}\left(Q_0^{\frac{2}{(\alpha_{pr}+\beta_{pr})}} + \frac{2}{(\alpha_{pr}+\beta_{pr})} Q_0^{\frac{2}{(\alpha_{pr}+\beta_{pr})}-1} \beta_S(P - P_0)\right), \tag{3.55}$$

$$TC(P) \approx \beta_{TC}(P - P_0) + TC_0 \tag{3.56}$$

where $\beta_{TC} = \frac{2\,\beta_S}{(\alpha_{pr}+\beta_{pr})}\frac{TC_0}{Q_0}$ and $TC_0 = c_{scale2}Q_0^{\frac{2}{(\alpha_{pr}+\beta_{pr})}}$.

Then the capital requirement and the labor requirement should be respectively

$$K(P) = \frac{\alpha_{pr}c_{scale2}}{r(\alpha_{pr} + \beta_{pr})}(\beta_{TC}(P - P_0) + TC_0) \tag{3.57}$$

and

$$L(P) = \frac{\beta_{pr}\,c_{scale2}}{w(\alpha_{pr} + \beta_{pr})}(\beta_{TC}(P - P_0) + TC_0) \tag{3.58}$$

from Eq. (3.36), Eq. (3.37) and Eq.(3.56). The labor quantity (L) is supplied from the labor market, and the capital quantity (K) is working as the investment spent in the commodity market. The total cost (TC) is mainly provided by loans in the finance market.

For an example, let the scale coefficient (c_{scale2}) is 1, and the factor contribution coefficients (α_{pr} and β_{pr}) have the relation of $\alpha_{pr} + \beta_{pr}$ = 0.8. Then, the supply curve ($S(P) = Q$) of Eq. (3.48) under the normal condition is plotted in Figure 8. Under the same condition, the labor demand and capital demand of the firm are plotted in Figure 9.

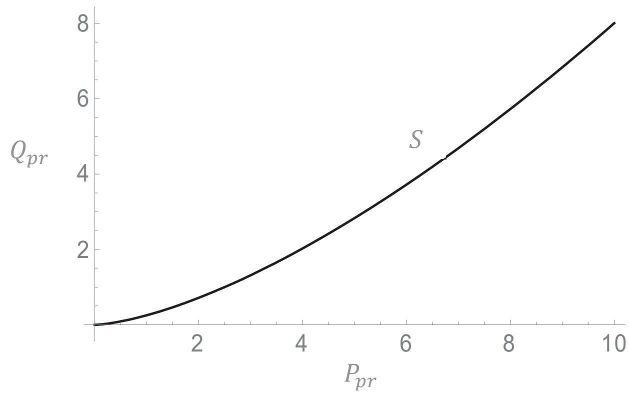


Figure 8. Supply curve of the firm with $\boldsymbol{c_{scale2}}$=1 and $\boldsymbol{\alpha_{pr} + \beta_{pr} = 0.8}$.

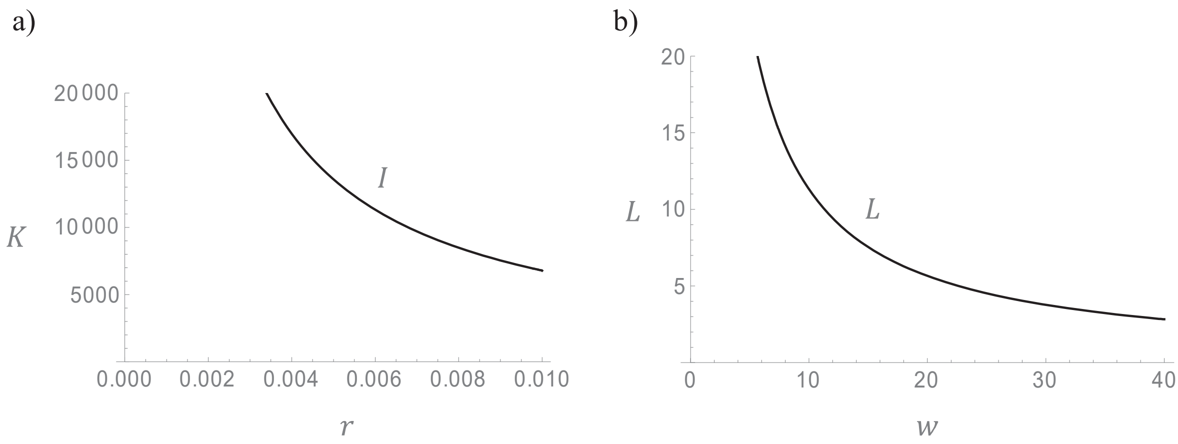


Figure 9. Firm's demand curves for a) capital and b) labors with $\boldsymbol{c_{scale2}}$ = 1, $\boldsymbol{\alpha_{pr}}$ = 0.3, $\boldsymbol{\beta_{pr} = 0.5}$ and P = 10.

Budget driven production

The firm can decide its production quantity or supply based on the budget. When the budget is given, the inserted quantities of the factors can be

$$K = \frac{\alpha_{pr}}{r(\alpha_{pr} + \beta_{pr})} Budget, \tag{3.59}$$

and

$$L = \frac{\beta_{pr}}{w(\alpha_{pr} + \beta_{pr})} Budget \tag{3.60}$$

from Eq. (3.36) and Eq. (3.37). The total production can be

$$Q = \frac{{\alpha_{pr}}^{\alpha_{pr}} {\beta_{pr}}^{\beta_{pr}}}{(\alpha_{pr} + \beta_{pr})^{(\alpha_{pr}+\beta_{pr})}} \frac{Budget^{(\alpha_{pr}+\beta_{pr})}}{r^{\alpha_{pr}} w^{\beta_{pr}}} \tag{3.61}$$

from Eq. (3.29). The firm will provide the commodity at the price of

$$P = \frac{1}{\beta_S}\left(\frac{{\alpha_{pr}}^{\alpha_{pr}} {\beta_{pr}}^{\beta_{pr}}}{(\alpha_{pr} + \beta_{pr})^{(\alpha_{pr}+\beta_{pr})}} \frac{Budget^{(\alpha_{pr}+\beta_{pr})}}{r^{\alpha_{pr}} w^{\beta_{pr}}} - Q_0\right) + P_0. \tag{3.62}$$

When the budget is bigger than that at the market equilibrium, this price from the firm will be higher than the market price at its equilibrium since over production makes the market price lower. So, the firm will suffer losses. Thus, the firm will not take this strategy, and the firm will decide the production quantity at the expected market price.

Correction of supply curve

So, the supply curve of Eq. (3.53) should be corrected with market expectation of the firm as

$$S(P) = \beta_S(P - P_0) + Q_0 + \beta_S^{\Delta p}\frac{dP}{dt} + \beta_S^{\Delta D}\frac{dD}{dt} \tag{3.63}$$

Where $\beta_S^{\Delta p}$ and $\beta_S^{\Delta D}$ are the firm's expectation coefficients respectively for price changes and for the demand changes. The firm cannot provide the quantity more than its production capacity, and it will provide no production from a long-run aspect when its average cost is higher than the marginal cost ($\frac{TC}{Q} > \frac{dTC}{dQ}$). From Eq. (3.47) and Eq. (3.48), possible production condition is

$$\alpha_{pr} + \beta_{pr} < 2. \tag{3.64}$$

So, the supply function will be

$$S(P) = \begin{cases} 0 & \alpha_{pr} + \beta_{pr} \geq 2 \\ S_{max} & S(P) > S_{max} \\ \beta_S(P - P_0) + Q_0 + \beta_S^{\Delta p}\frac{dP}{dt} + \beta_S^{\Delta D}\frac{dD}{dt} & else \end{cases} \tag{3.65}$$

where S_{max} is the production capacity of the firm.

3.2.3 Government

The government acts like a market moderator. It acts independently without any strong constraint in the markets. In the commodity market, the government can increase or decrease its expenditure to increase or decrease the demand, and it can increase or decrease taxes on the consumers or the firms to decrease or increase their demand. In finance market, the government can sell or buy treasury bonds to decrease or increase money flow. The government can also control the base interest rate to control savings and loans. In this section, no action equation of the government is derived since government has no constraint. Rather, its action will be discussed in the market sections.

3.2.4 Foreign countries

Foreign countries can participate in the finance market or in the commodity market of a country through the exchange market due to the restricted mobility of labor. Their activities basically follow the principles of trade. The forces of the trade are the differences in prices cross countries. In finance markets, the price will be the interest rate of a country. Here the brief mathematical description about trade will followed.

What makes countries do trade?

Trade means merging the commodity market of the countries. Let assume the trade between two countries; country A imports some commodity from country B, and country B exports the commodity to country A. The commodity market of each country has its own demand curve and supply curve, and those are assumed to be expressed as simple linear equation forms as

$$D^A(p) = -\alpha^A\,(p - p_0^A) + Q_0^A \tag{3.66}$$

$$S^A(p) = \beta^A\,(p - p_0^A) + Q_0^A \tag{3.67}$$

$$D^B(p) = -\alpha^B\,(p - p_0^B) + Q_0^B \tag{3.68}$$

$$S^B(p) = \beta^B\,(p - p_0^B) + Q_0^B \tag{3.69}$$

where $D(p)$, $S(p)$, α, β, p, p_0, Q_0 and superscript are demand and supply, demand elasticity, supply elasticity, price, equilibrium price, equilibrium quantity and country name. After commodity trade, the demand curves and supply curves would be

$$D^A(p)_{after\ trade} = D^A(p) = -\alpha^A\ (p - p_0^A) + Q_0^A \tag{3.70}$$

$$S^A(p)_{after\ trade} = \beta^A\ (p - p_0^A) + Q_0^A + Q_{im} \tag{3.71}$$

$$D^B(p)_{after\ trade} = D^B(p) = -\alpha^B\ (p - p_0^B) + Q_0^B \tag{3.72}$$

$$S^B(p)_{after\ trade} = \beta^B\ (p - p_0^B) + Q_0^B - Q_{ex} \tag{3.73}$$

Where Q_{im} and Q_{ex} are respectively imported quantity and exported quantity. And new equilibrium prices are

$$p_0^A{}_{after\ trade} = p_0^A - \frac{Q_{im}}{\alpha^A + \beta^A} \tag{3.74}$$

$$p_0^B{}_{after\ trade} = p_0^B + \frac{Q_{ex}}{\alpha^B + \beta^B} \tag{3.75}$$

using Eq. (3.70) = Eq. (3.71) and Eq. (3.72) = Eq. (3.73).

After full trade between country A and country B, two commodity markets are merged, and the prices should be identical. So, $p_0^B{}_{after\ trade} = p_0^A{}_{after\ trade}$ and

$$Q_{trade} = \frac{p_0^A - p_0^B}{\frac{1}{\alpha^A + \beta^A} + \frac{1}{\alpha^B + \beta^B}} \tag{3.76}$$

where $Q_{trade} = Q_{im} = Q_{ex}$. Eq. (3.76) reveals the fundamental reason of the trade. The price difference makes the trade possible. So, when the prices of two countries are same, they do not trade even if they have different abundant resource.

$$p_0^A{}_{after\ trade} = p_0^A - \frac{Q_{trade}}{\alpha^A + \beta^A} = p_0^A - (p_0^A - p_0^B)\left(\frac{\alpha^B + \beta^B}{\alpha^A + \beta^A + \alpha^B + \beta^B}\right) \quad (3.77)$$

$$p_0^B{}_{after\ trade} = p_0^B + \frac{Q_{trade}}{\alpha^B + \beta^B} = p_0^B + (p_0^A - p_0^B)\left(\frac{\alpha^A + \beta^A}{\alpha^A + \beta^A + \alpha^B + \beta^B}\right) \quad (3.78)$$

Eq. (3.77) and Eq. (3.78) tell us what the trade effect is. Two countries can get the portion of the price difference $(p_0^A - p_0^B)$ by the price elasticity through the trade. Thus trade refers to be the price adjustment through market merging.

Trades in finance market act as likely as those in commodity market. And the interest rate (r_0^A and r_0^B) and money volume ($M_{trade}^{\$}$) can be treated as the price of money and the trade quantity of money, respectively. So, the interest rates of the countries after financial trades can be expressed as

$$r_0^A{}_{after\ trade} = r_0^A - \frac{ex_{\$}^A\ M_{trade}^{\$}}{\alpha_r^A + \beta_r^A} \quad (3.79)$$

and

$$r_0^B{}_{after\ trade} = r_0^B + \frac{ex_{\$}^B\ M_{trade}^{\$}}{\alpha_r^B + \beta_r^B} \quad (3.80)$$

where $ex_{\$}^A$ and $ex_{\$}^B$ are exchange rates to USD of Country A and Country B, respectively.

The traded money volume will be

$$M_{trade}^{\$} = \frac{r_0^A - r_0^B}{\dfrac{ex_{\$}^A}{\alpha_r^A + \beta_r^A} + \dfrac{ex_{\$}^B}{\alpha_r^B + \beta_r^B}} \quad (3.81)$$

in USD unites. The interest rate after the financial trade will be

$$r_{0\ after\ trade}^{A} = r_0^A - (r_0^A - r_0^B)\left(\frac{\alpha_r^B + \beta_r^B}{ex_{\$}^A(\alpha_r^B + \beta_r^B) + ex_{\$}^B(\alpha_r^A + \beta_r^A)}\right), \tag{3.82}$$

or

$$r_{0\ after\ trade}^{B} = r_0^B + (r_0^A - r_0^B)\left(\frac{\alpha_r^A + \beta_r^A}{ex_{\$}^A(\alpha_r^B + \beta_r^B) + ex_{\$}^B(\alpha_r^A + \beta_r^A)}\right). \tag{3.83}$$

3.3 Markets

Markets match consumer's demands and provider's supplies, and it determines the price levels for their trades. This section covers the 4 markets (commodity market, labor market finance market and exchange market). Though they deal with different items for trades, their mechanisms are identical. Most economic books concern just its equilibrium, and the transient dynamics of a market is rarely described. However, real markets seem to be more dynamic since their situations or conditions are continuously changing before reaching their equilibriums. So, this section will deal with the common market dynamics more rigorously and more mathematically.

3.3.1 Market dynamics

In normal cases, the demand curve and supply curve can be

$$D(P) = -\alpha_D P + D_{P=0} + \alpha_D^{\Delta P} \frac{dP}{dt} + \alpha_D^{\Delta S} \frac{dS}{dt} \tag{3.84}$$

$$S(P) = \beta_S P + S_{P=0} + \beta_S^{\Delta P} \frac{dP}{dt} + \beta_S^{\Delta D} \frac{dD}{dt} \tag{3.85}$$

from Eq. (3.21) and Eq. (3.55). And Eq. (3.84) and Eq. (3.85) would be

$$D(P) = -\alpha_D P + D_{P=0} + \alpha_D^{\Delta P} \frac{dP}{dt} + \alpha_D^{\Delta S} \frac{d}{dt}\left(\beta_S P + S_{P=0} + \beta_S^{\Delta P} \frac{dP}{dt} + \beta_S^{\Delta D} \frac{dD}{dt}\right) \tag{3.86}$$

$$= -\alpha_D P + D_{P=0} + \left(\alpha_D^{\Delta P} + \alpha_D^{\Delta S}\beta_S\right)\frac{dP}{dt} + \alpha_D^{\Delta S}\beta_S^{\Delta P}\frac{d^2P}{dt^2} + \alpha_D^{\Delta S}\beta_S^{\Delta D}\frac{d^2D}{dt^2} \tag{3.87}$$

$$S(P) = \beta_S P + S_{p=0} + \beta_S^{\Delta P}\frac{dP}{dt} + \beta_S^{\Delta D}\frac{d}{dt}\left(-\alpha_D P + D_{P=0} + \alpha_D^{\Delta P}\frac{dP}{dt} + \alpha_D^{\Delta S}\frac{dS}{dt}\right) \tag{3.88}$$

$$= \beta_S P + S_{p=0} + \left(\beta_S^{\Delta p} - \alpha_D\beta_S^{\Delta D}\right)\frac{dP}{dt} + \alpha_D^{\Delta P}\beta_S^{\Delta D}\frac{d^2P}{dt^2} + \alpha_D^{\Delta S}\beta_S^{\Delta D}\frac{d^2S}{dt^2} \tag{3.89}$$

Net demand (ND) can be defined as

$$ND(P) \equiv D(P) - S(P), \tag{3.90}$$

and it can be

$$ND(P) = -(\alpha_D + \beta_S)P + D_{P=0} - S_{p=0} + \left(\alpha_D^{\Delta P} - \beta_S^{\Delta P} + \alpha_D^{\Delta S}\beta_S + \alpha_D\beta_S^{\Delta D}\right)\frac{dP}{dt} + \left(\alpha_D^{\Delta S}\beta_S^{\Delta P} - \alpha_D^{\Delta P}\beta_S^{\Delta D}\right)\frac{d^2P}{dt^2} + \alpha_D^{\Delta S}\beta_S^{\Delta D}\frac{d^2ND}{dt^2} \tag{3.91}$$

from Eq. (3.87) and Eq. (3.89). To solve the differential equation, Eq. (3.91) can be arranged as

$$ND(P) - \alpha_D^{\Delta S}\beta_S^{\Delta D}\frac{d^2ND}{dt^2} - ND_{P=0} = -(\alpha_D + \beta_S)P \tag{3.92}$$

$$+\left(\alpha_D^{\Delta P} - \beta_S^{\Delta P} + \alpha_D \beta_S^{\Delta D} + \alpha_D^{\Delta S} \beta_S\right) \frac{dP}{dt}$$

$$+(\alpha_D^{\Delta S} \beta_S^{\Delta P} - \alpha_D^{\Delta P} \beta_S^{\Delta D}) \frac{d^2 P}{dt^2}$$

where $ND_{P=0} = D_{P=0} - S_{p=0}$.

To solve the differential equation of Eq. (3.92), let

$$P = \Delta P_0 \, e^{\Omega t} + P_0 \tag{3.93}$$

where ΔP, Ω and P_0 are respectively the price change amplitude, the price oscillation frequency and the equilibrium price.

The homogenous differential equation part of Eq. (3.92) can be

$$\left(\alpha_D^{\Delta S} \beta_S^{\Delta P} - \alpha_D^{\Delta P} \beta_S^{\Delta D}\right)\Omega^2 + \left(\alpha_D^{\Delta P} - \beta_S^{\Delta P} + \alpha_D \beta_S^{\Delta D} + \alpha_D^{\Delta S} \beta_S\right)\Omega - (\alpha_D + \beta_S) = 0 \tag{3.94}$$

after insertion of Eq. (3.93) into Eq. (3.92), and the price oscillation frequency (Ω) can be found as

$$\Omega = \frac{-\left(\alpha_D^{\Delta P} - \beta_S^{\Delta P} + \alpha_D \beta_S^{\Delta D} + \alpha_D^{\Delta S} \beta_S\right)}{2\left(\alpha_D^{\Delta S} \beta_S^{\Delta P} - \alpha_D^{\Delta P} \beta_S^{\Delta D}\right)} \pm \frac{\sqrt{\left(\alpha_D^{\Delta P} - \beta_S^{\Delta P} + \alpha_D \beta_S^{\Delta D} + \alpha_D^{\Delta S} \beta_S\right)^2 + 4(\alpha_D + \beta_S)\left(\alpha_D^{\Delta S} \beta_S^{\Delta P} - \alpha_D^{\Delta P} \beta_S^{\Delta D}\right)}}{2\left(\alpha_D^{\Delta S} \beta_S^{\Delta P} - \alpha_D^{\Delta P} \beta_S^{\Delta D}\right)} \tag{3.95}$$

and it can be a form of

$$\Omega = -\kappa + \mathrm{i}\,\varpi \tag{3.96}$$

where

$$\kappa = \frac{\alpha_D^{\Delta P} - \beta_S^{\Delta P} + \alpha_D \beta_S^{\Delta D} + \alpha_D^{\Delta S} \beta_S}{2\left(\alpha_D^{\Delta S} \beta_S^{\Delta P} - \alpha_D^{\Delta P} \beta_S^{\Delta D}\right)} \tag{3.97}$$

and

$$\varpi = \frac{\sqrt{4(\alpha_D + \beta_S)\left(\alpha_D^{\Delta P} \beta_S^{\Delta D} - \alpha_D^{\Delta S} \beta_S^{\Delta P}\right) - \left(\alpha_D^{\Delta P} - \beta_S^{\Delta P} + \alpha_D \beta_S^{\Delta D} + \alpha_D^{\Delta S} \beta_S\right)^2}}{2\left(\alpha_D^{\Delta S} \beta_S^{\Delta P} - \alpha_D^{\Delta P} \beta_S^{\Delta D}\right)}. \tag{3.98}$$

κ and ϖ means respectively the decay coefficient and the oscillation frequency of market price.

Including the inhomogeneous part of Eq. (3.92), Eq. (3.92) can be

$$-(\alpha_D + \beta_S)(\Delta P_0 + P_0) = ND_{t=0} - ND_{P=0} \tag{3.99}$$

at t = 0 since $\frac{dP}{dt} = \frac{d^2P}{dt^2} = \left.\frac{d^2ND}{dt^2}\right|_{t=0} = 0$, and Eq. (3.92) can be also

$$-(\alpha_D + \beta_S)P_0 = ND_{t=\infty} + \alpha_D^{\Delta S} \beta_S^{\Delta D} \left.\frac{d^2ND}{dt^2}\right|_{t=\infty} - ND_{P=0} \tag{3.100}$$

at $t = \infty$ since $\frac{dP}{dt} = \frac{d^2P}{dt^2} = 0$.

P_0 will be

$$P_0 = \frac{ND_{P=0}}{\alpha_D + \beta_S} \tag{3.101}$$

since $ND_{t=\infty}$=0 and $\left.\frac{d^2ND}{dt^2}\right|_{t=\infty} = 0$. ΔP_0 can be

$$\Delta P_0 = \frac{ND_{t=0}}{-\left(\alpha_D + \beta_S\right)} \tag{3.102}$$

from Eq. eq-m-14 and Eq. eq-m-16.

Now, Eq. (3.93) is fully described with Eq. (3.95), Eq. (3.101) and Eq. (3.102), and the price changes in a market are described in Figure 10, fig-m-2 and fig-m-3. Figure 10 shows the general price adjustments to resolve discrepancy between demands and supplies. When the price prediction of the consumers' is extremely wrong or when the panic behaviors fail the price adjustment of the market in as shown in Figure 12 and 13.

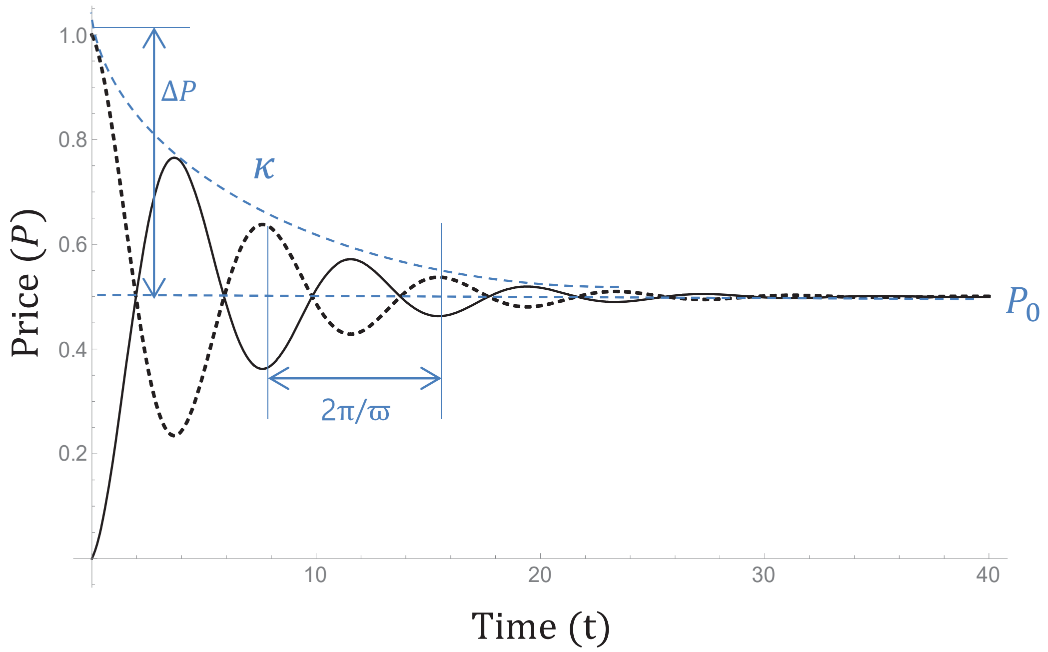


Figure 10. Price adjustment by market when $\boldsymbol{\alpha_D = 1, \beta_S = 1, \alpha_D^{\Delta P} = 1, \beta_S^{\Delta P} = 2, \alpha_D^{\Delta S} = -1, \beta_S^{\Delta D} = 1, ND_{P=0} = 1}$ **and** $\boldsymbol{ND_{t=0} = 1}$ (solid line), $\boldsymbol{-1}$ (dash line).

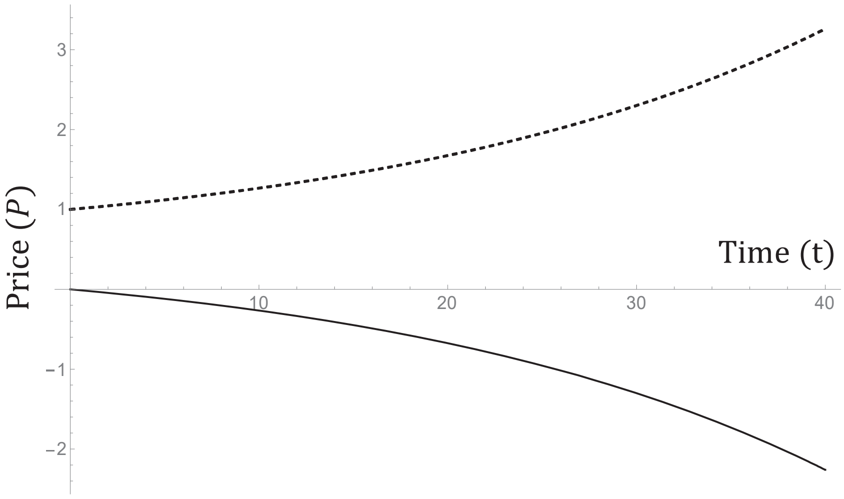


Figure 12. Price divergence by consumers' panic behavior when $\boldsymbol{\alpha_D = 1, \beta_S = 1, \alpha_D^{\Delta P} = 50, \beta_S^{\Delta P} = 1, \alpha_D^{\Delta S} = -1, \beta_S^{\Delta D} = 1, ND_{P=0} = 1}$ **and** $\boldsymbol{ND_{t=0} = 1}$ (solid line), $\boldsymbol{-1}$ (dash line).

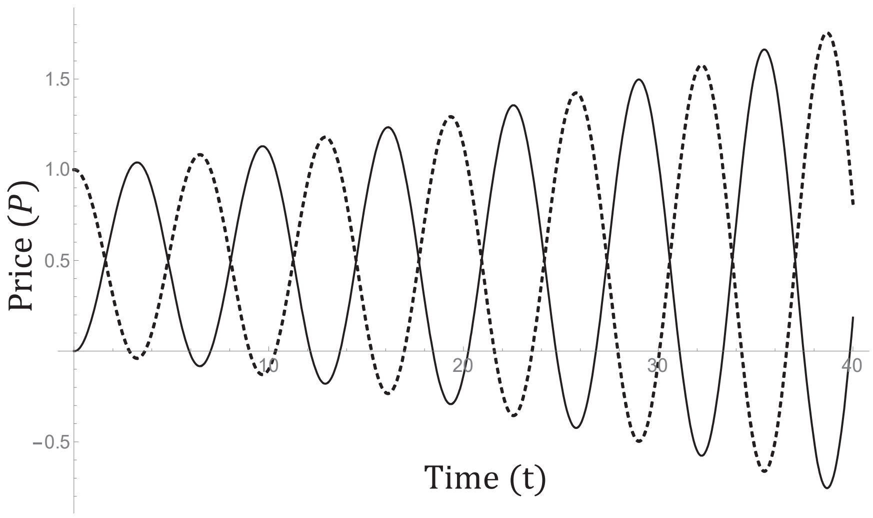


Figure 13. Oscillating price divergence by consumers' panic behavior when $\boldsymbol{\alpha_D = 1, \beta_S = 1, \alpha_D^{\Delta P} = 1.1, \beta_S^{\Delta P} = 1, \alpha_D^{\Delta S} = -1, \beta_S^{\Delta D} = 1, ND_{P=0} = 1}$ **and** $\boldsymbol{ND_{t=0} = 1}$ (solid line), $\boldsymbol{-1}$ (dash line)

3.3.2 Commodity market

The total demand of the commodity market can be expressed as

$$D_{monetary}^{tot} = C + I + G + NX \tag{3.103}$$

in monetary value from Eq. eq-st-1 as shown in Figure 6. The demand from house holders (C) will be

$$C(P) = PQ_D = P\left(-\alpha_D P + Q_c|_{P=0} + \alpha_D^{\Delta P}\frac{dP}{dt} + \alpha_D^{\Delta S}\frac{dS}{dt}\right) \tag{3.104}$$

around P_0 from Eq. eq-c-17, and the coefficient α_D and $Q_c|_{P=0}$ are the function ofs net income as

$$\alpha_D = \frac{\beta_c\, W_{net}(P,r)}{(\alpha_c + \beta_c + \gamma_c\, r){P_0}^2} \tag{3.105}$$

and

$$Q_c|_{P=0} = \frac{2\beta_c\, W_{net}(P,r)}{(\alpha_c + \beta_c + \gamma_c\, r)P_0}. \tag{3.106}$$

The net income is derived to be a function of price (P) and interest rate (r) from Eq. (3.21) and Eq. (3.146) as

$$W_{net}(P,r) = w(P)L(P) - S_c(P,r) \tag{3.107}$$

$$\approx \frac{\beta_{pr}\, c_{scale2}}{\left(\alpha_{pr} + \beta_{pr}\right)}\left((P - P_0) + TC_0\right) - \left(\alpha_{S_c}(r - r_0) + S_{C0}\right) \tag{3.108}$$

$$= \beta_{W_{net}}^{P}(P - P_0) - \beta_{W_{net}}^{r}(r - r_0) + W_{net0} \tag{3.109}$$

where

$$\beta_{W_{net}}^{P} = \frac{\beta_{pr}\ c_{scale2}}{(\alpha_{pr} + \beta_{pr})} \tag{3.110}$$

$$\beta_{W_{net}}^{r} = \alpha_{S_c}, \tag{3.111}$$

and

$$W_{net0} = \frac{\beta_{pr}\ c_{scale2}}{(\alpha_{pr} + \beta_{pr})} TC_0 - S_{C0} \tag{3.112}$$

around equilibrium. So, the net income is directly proportional to the price level (P) and the interest rate (r). However, the net income will be treated as a given constant value (W_{net0}) due to the price rigidity of the income. The demand from the firms of Figure 6 can be written as

$$I(P) = K(P) = \frac{c_{scale3}}{r}(\beta_{TC}P + TC_0|_{P=0}) + \beta_{TC}^{\Delta P}\frac{dP}{dt} + \beta_{TC}^{\Delta D}\frac{dD}{dt} \tag{3.113}$$

where $c_{scale3} = \frac{\alpha_{pr}c_{scale2}}{\alpha_{pr}+\beta_{pr}}$ from Eq. (3.57) with firm's predictions. The demand induced by the export of Figure 6 is

$$NX = P\ Q_{trade} = P\frac{P^{foreign} - P}{\frac{1}{a} + \frac{1}{a^{foreign}}} \tag{3.114}$$

where $a = \alpha_D + \beta_S,\ a^{foreign} = \alpha_D^{foreign} + \beta_S^{foreign}$ from Eq. (3.76). The demand induced by the expenditure of the government is

$$G(P, r) = P * Q_G \tag{3.115}$$

from Eq. (3.76) in monetary value. So, the total demand of Eq. (3.103) is

$$D_{monetary}^{tot} = P * D(P)$$

$$= P\left(-\alpha_D P + Q_c|_{P=0} + \alpha_D^{\Delta P}\frac{dP}{dt} + \alpha_D^{\Delta S}\frac{dS}{dt}\right)$$

$$+ \left(\frac{c_{scale3}}{r}(\beta_{TC}P + TC_0|_{P=0}) + \beta_{TC}^{\Delta P}\frac{dP}{dt} + \beta_{TC}^{\Delta D}\frac{dD}{dt}\right) + P * Q_G \qquad (3.116)$$

$$+ P\frac{P^{foreign} - P}{\frac{1}{a} + \frac{1}{a^{foreign}}},$$

and the total supply from the firm in the commodity market is

$$S_{monetary}^{tot} = P * S(P) = P\left(\beta_S P + Q_s|_{P=0} + \beta_S^{\Delta P}\frac{dP}{dt} + \beta_S^{\Delta D}\frac{dD}{dt}\right). \qquad (3.117)$$

The net demand will be

$$ND_{monetary}^{tot} = P\left(-\alpha_D P + Q_c|_{P=0} + \alpha_D^{\Delta P}\frac{dP}{dt} + \alpha_D^{\Delta S}\frac{dS}{dt}\right)$$

$$+ \left(\frac{c_{scale3}}{r}(\beta_{TC}P + TC_0|_{P=0}) + \beta_{TC}^{\Delta P}\frac{dP}{dt} + \beta_{TC}^{\Delta D}\frac{dD}{dt}\right) + P * Q_G \qquad (3.118)$$

$$+ P\frac{P^{foreign} - P}{\frac{1}{a} + \frac{1}{a^{foreign}}} - P\left(\beta_S P + Q_s|_{P=0} + \beta_S^{\Delta P}\frac{dP}{dt} + \beta_S^{\Delta D}\frac{dD}{dt}\right),$$

Eq. (3.118) can be rewritten in quantity value as

$$ND = \left(\alpha_D^{\Delta p} - \beta_S^{\Delta p} + \frac{\beta_{TC}^{\Delta P}}{P}\right)\frac{dP}{dt} - \left(\alpha_D + \beta_S - \frac{c_{scale3}}{r\,P}\beta_{TC} + \frac{1}{\frac{1}{a} + \frac{1}{a^{foreign}}}\right)P$$

$$+ Q_c|_{P=0} - Q_s|_{P=0} + Q_G + \frac{c_{scale3}}{r\,P}TC_0|_{P=0} + \frac{P^{foreign}}{\frac{1}{a} + \frac{1}{a^{foreign}}} \qquad (3.119)$$

$$+ \alpha_D^{\Delta S}\frac{dS}{dt} + \left(\beta_{TC}^{\Delta D} - \beta_S^{\Delta D}\right)\frac{dD}{dt}$$

where $ND(P) \equiv D(P) - S(P)$.

Eq. (3.119) has no analytic solution though it can be solved numerically. Considering slow movements of investments, Eq. (3.119) can be modified to have it analytic solution as

$$\begin{aligned}
&\left(\alpha_D^{\Delta S}\beta_S^{\Delta P} + \alpha_D^{\Delta P}\left(\beta_{TC}^{\Delta D} - \beta_S^{\Delta D}\right) + \left(\beta_{TC}^{\Delta D} - \beta_S^{\Delta D}\right)\frac{\beta_{TC}^{\Delta P}}{P_0}\right)\frac{d^2P}{dt^2} \\
&\quad + \left(\alpha_D^{\Delta p} - \beta_S^{\Delta p} + \frac{\beta_{TC}^{\Delta P}}{P_0} + \alpha_D^{\Delta S}\beta_S \right. \\
&\quad \left. + \left(\frac{c_{scale3}\beta_{TC}}{r\,P_0} - \alpha_D - \frac{1}{\frac{1}{a} + \frac{1}{a^{foreign}}}\right)\left(\beta_{TC}^{\Delta D} - \beta_S^{\Delta D}\right)\right)\frac{dP}{dt} \\
&\quad - \left(\alpha_D + \beta_S - \frac{c_{scale3}}{r\,P_0}\beta_{TC} + \frac{1}{\frac{1}{a} + \frac{1}{a^{foreign}}}\right)P + Q_c|_{P=0} \\
&\quad - Q_s|_{P=0} + Q_G + \frac{c_{scale3}}{r\,P_0}TC_0|_{P=0} + \frac{P^{foreign}}{\frac{1}{a} + \frac{1}{a^{foreign}}} \\
&\quad + \left(\beta_{TC}^{\Delta D} - \beta_S^{\Delta D}\right)\frac{1}{\frac{1}{a} + \frac{1}{a^{foreign}}}\frac{dP^{foreign}}{dt} - ND = 0
\end{aligned} \tag{3.120}$$

due to slow varying aggregated supply and demand ($\frac{d^2D}{dt^2} = \frac{d^2S}{dt^2} = 0$).

The solution of Eq. (3.120) can be set as $P = \triangle P_0\, e^{\Omega t}$ for its homogenous differential part :

$$\left(\alpha_D^{\Delta S}\beta_S^{\Delta P} + \alpha_D^{\Delta P}\left(\beta_{TC}^{\Delta D} - \beta_S^{\Delta D}\right) + \left(\beta_{TC}^{\Delta D} - \beta_S^{\Delta D}\right)\frac{\beta_{TC}^{\Delta P}}{P_0}\right)\Omega^2$$
$$+\left(\alpha_D^{\Delta p} - \beta_S^{\Delta p} + \frac{\beta_{TC}^{\Delta P}}{P_0} + \alpha_D^{\Delta S}\beta_S\right.$$
$$\left. + \left(\frac{c_{scale3}\beta_{TC}}{r\,P_0} - \alpha_D - \frac{1}{\frac{1}{a} + \frac{1}{a^{foreign}}}\right)\left(\beta_{TC}^{\Delta D} - \beta_S^{\Delta D}\right)\right)\Omega \tag{3.121}$$
$$-\left(\alpha_D + \beta_S - \frac{c_{scale3}}{r\,P}\beta_{TC} + \frac{1}{\frac{1}{a} + \frac{1}{a^{foreign}}}\right) = 0.$$

Then Ω will be

$$\Omega = -\kappa + \mathrm{i}\,\omega \tag{3.122}$$

where

$$\kappa = \frac{\alpha_D^{\Delta p} - \beta_S^{\Delta p} + \frac{\beta_{TC}^{\Delta P}}{P} + \alpha_D^{\Delta S}\beta_S + \left(\frac{c_{scale3}\beta_{TC}}{r\,P_0} - \alpha_D - \frac{1}{\frac{1}{a} + \frac{1}{a^{foreign}}}\right)\left(\beta_{TC}^{\Delta D} - \beta_S^{\Delta D}\right)}{2\left(\alpha_D^{\Delta S}\beta_S^{\Delta P} + \alpha_D^{\Delta P}\left(\beta_{TC}^{\Delta D} - \beta_S^{\Delta D}\right) + \left(\beta_{TC}^{\Delta D} - \beta_S^{\Delta D}\right)\frac{\beta_{TC}^{\Delta P}}{P_0}\right)} \tag{3.123}$$

and

$$\omega = \left(\frac{\alpha_D + \beta_S - \frac{c_{scale3}}{r\,P_0}\beta_{TC} + \frac{1}{\frac{1}{a} + \frac{1}{a^{foreign}}}}{\alpha_D^{\Delta P}\left(\beta_S^{\Delta D} - \beta_{TC}^{\Delta D}\right) + \left(\beta_S^{\Delta D} - \beta_{TC}^{\Delta D}\right)\frac{\beta_{TC}^{\Delta P}}{P_0} - \alpha_D^{\Delta S}\beta_S^{\Delta P}}\right. \tag{3.124}$$

$$
-\left(\frac{\alpha_D^{\Delta p} - \beta_S^{\Delta p} + \frac{\beta_{TC}^{\Delta P}}{P_0} + \alpha_D^{\Delta S}\beta_S + \left(\frac{c_{scale3}\beta_{TC}}{r\,P_0} - \alpha_D - \frac{1}{\frac{1}{a} + \frac{1}{a^{foreign}}}\right)\left(\beta_{TC}^{\Delta D} - \beta_S^{\Delta D}\right)}{4\left(\alpha_D^{\Delta P}\left(\beta_S^{\Delta D} - \beta_{TC}^{\Delta D}\right) + \left(\beta_S^{\Delta D} - \beta_{TC}^{\Delta D}\right)\frac{\beta_{TC}^{\Delta P}}{P_0} - \alpha_D^{\Delta S}\beta_S^{\Delta P}\right)}\right)^{2}\Bigg)^{1/2}
$$

For in-homogenous equation of Eq. (3.120), the solution can be set as

$$
P = \triangle P_0\, e^{\Omega\, t} + P_0\,. \tag{3.125}
$$

When $t = \infty$, Eq. (3.120) will be

$$
\left(\alpha_D + \beta_S + \frac{1}{\frac{1}{a} + \frac{1}{a^{foreign}}}\right){P_0}^2
- \left(Q_c|_{P=0} - Q_s|_{P=0} + Q_G + \frac{c_{scale3}}{r}\beta_{TC} + \frac{p^{foreign}}{\frac{1}{a} + \frac{1}{a^{foreign}}}\right)P_0
- \frac{c_{scale3}}{r}TC_0|_{P=0} = 0. \tag{3.126}
$$

When we let $C_a = \alpha_D + \beta_S + \frac{1}{\frac{1}{a}+\frac{1}{a^{foreign}}}$, $C_b = Q_c|_{P=0} - Q_s|_{P=0} + Q_G + \frac{c_{scale3}}{r}\beta_{TC} + \frac{p^{foreign}}{\frac{1}{a}+\frac{1}{a^{foreign}}}$ and $C_c = \frac{c_{scale3}}{r}TC_0|_{P=0}$. The equilibrium price (P_0) can be

$$
P_0 = \frac{C_b \pm \sqrt{{C_b}^2 + 4C_aC_c}}{2\,C_a} \approx \frac{C_b}{2\,C_a}\left(1 \pm \left(1 + \frac{2C_aC_c}{{C_b}^2}\right)\right) => \frac{C_b}{2\,C_a}\left(2 + \frac{2C_aC_c}{{C_b}^2}\right) \tag{3.127}
$$

after the solution selection of positive P_0, and it can be simplified as

$$
P_0 = \frac{C_b}{C_a} + \frac{C_c}{C_b}. \tag{3.128}
$$

When $t = 0$, Eq. (3.120) can be

$$(C_a)(\triangle P_0 + P_0)$$

$$= C_b + C_c + \left(\beta_{TC}^{\Delta D} - \beta_S^{\Delta D}\right) \frac{1}{\frac{1}{a} + \frac{1}{a^{foreign}}} \frac{dP^{foreign}}{dt}\bigg|_{t=0} \tag{3.129}$$

$$- ND_{t=0}$$

The oscillation amplitude ($\triangle P_0$) will be

$$\triangle P_0 = \frac{C_c}{C_a} - \frac{C_c}{C_b} + \left(\beta_{TC}^{\Delta D} - \beta_S^{\Delta D}\right) \frac{1}{\frac{1}{a} + \frac{1}{a^{foreign}}} \frac{dP^{foreign}}{dt}\bigg|_{t=0} - ND_{t=0}, \tag{3.130}$$

and it can be rewritten as

$$\triangle P_0 = -\frac{ND_{t=0} - \left(\frac{\beta_{TC}^{\Delta D} - \beta_S^{\Delta D}}{\frac{1}{a} + \frac{1}{a^{foreign}}}\right) \frac{dP^{foreign}}{dt}\bigg|_{t=0} - \frac{c_{scale3}}{r} TC_0|_{P=0}}{C_a} - \frac{C_c}{C_b}. \tag{3.131}$$

Now, the price function is fully derived and it will show the behavior in Figure 10 though it has the modifications with the interest rate (r) and the price of the exported country.

The total yield (Y) in Figure 6 can be derived as

$$Y = P * S(P) = P\left(\beta_S P + Q_s|_{P=0} + \beta_S^{\Delta P} \frac{dP}{dt} + \beta_S^{\Delta D} \frac{dD}{dt}\right) \tag{3.132}$$

$$= P\left(\beta_S P + Q_s|_{P=0} + \left(\beta_S^{\Delta P} - \beta_S^{\Delta D}\left(\alpha_D - \frac{c_{scale3}}{rP}\beta_{TC} + \frac{1}{\frac{1}{a} + \frac{1}{a^{foreign}}}\right)\right)\frac{dP}{dt} + \beta_S^{\Delta D}\left(\alpha_D^{\Delta P} + \frac{\beta_{TC}^{\Delta P}}{P}\right)\frac{d^2P}{dt^2}\right) \quad (3.133)$$

The total yield or GDP of a country can be expressed as

$$Y_0 = P_0\,(\beta_S P_0 + Q_s|_{P=0}) \quad (3.134)$$

after the stabilization of the market, and the GDP turns out to have a quadratic relation to the price level. When Eq. (3.128) is applied to Eq. (3.133), the GDP of a country can be described as

$$Y_0 = \beta_S\left(\frac{C_b}{C_a} + \frac{C_c}{C_b}\right)^2 + Q_s|_{P=0}\left(\frac{C_b}{C_a} + \frac{C_c}{C_b}\right) \quad (3.135)$$

where $C_a = \alpha_D + \beta_S + \frac{1}{\frac{1}{a}+\frac{1}{a^{foreign}}}$, $C_b = Q_c|_{P=0} - Q_s|_{P=0} + Q_G + \frac{c_{scale3}}{r}\beta_{TC} + \frac{P^{foreign}}{\frac{1}{a}+\frac{1}{a^{foreign}}}$ and $C_c = \frac{c_{scale3}}{r}TC_0|_{P=0}$. So, total yield or GDP of a country is totally dependent on the government expenditure (Q_G), the price level of its trade countries ($P^{foreign}$) and interest rate (r) since the net income ($W_{net}(P,r)$) in $Q_c|_{P=0}$ of C_b is also controlled by price level (P) and interest rate (r). Considering that the foreign price level can be manipulated by exchange rate, the exchange rate (e), the interest rate (r) and the government expenditure (Q_G) are the main actuators to run the economics of a country.

3.3.3 Labor market

When the commodity market determines the product price, the labor demand (L_D) from the firm was derived as Eq. (3.58) without any prediction behaviors, and it is rewritten as

$$L_D(P,w) = \frac{\beta_{pr}\, c_{scale2}}{w(\alpha_{pr} + \beta_{pr})}(\beta_{TC}(P - P_0) + TC_0) \tag{3.136}$$

while the labor supply (L_S) from the house holder is fixed as

$$L_S(P,w) = L_{S0}. \tag{3.137}$$

So, the unemployment (UE) can be derived as

$$UE = L_S(P,w) - L_D(P,w) = L_{S0} - \frac{\beta_{pr}\, c_{scale2}}{w(\alpha_{pr}+ \beta_{pr})}(\beta_{TC}(P - P_0) + TC_0), \tag{3.138}$$

and the wage will be

$$w(P) = \frac{\beta_{pr}\, c_{scale2}}{(L_{S0}-UE)(\alpha_{pr}+ \beta_{pr})}(\beta_{TC}(P - P_0) + TC_0). \tag{3.139}$$

Under the complete employment, the wage of the labor will be

$$w(P) = \frac{\beta_{pr}\, c_{scale2}}{L_{S0}(\alpha_{pr} + \beta_{pr})}(\beta_{TC}(P - P_0) + TC_0) \tag{3.140}$$

or

$$w(t) = \frac{\triangle W_0}{(L_{S0} - UE(t))}\, e^{\Omega\, t} + w_0 \tag{3.141}$$

where $\triangle W_0 = \frac{\beta_{pr}\, c_{scale2}}{(\alpha_{pr}+ \beta_{pr})}\beta_{TC} \triangle P_0$ and $w_0 = \frac{\beta_{pr}\, c_{scale2}}{L_{S0}(\alpha_{pr}+ \beta_{pr})}TC_0$. Eq. (3.141) will be

$$w(t) = \frac{\triangle W_0}{(L_{S0} - UE(t))}\, e^{\Omega\, t} + w_0 \tag{3.142}$$

since $UE(\infty) = 0$.

Considering that the labor demand from firms does not simultaneously follow the price change, Eq. (3.136) should be also modified to have a retardation time as

$$L_D(t) = \frac{\triangle W_0}{w(t)}\ e^{\Omega\,(t-t_{L_D})} + L_{D0} \tag{3.143}$$

where t_{L_D} is the retardation time of the labor demand for price change, and $L_{D0} = \frac{TC_0\beta_{pr}\,c_{scale2}}{(\alpha_{pr}+\beta_{pr})w_0} = L_{S0}$. The final equation (Eq. (3.142)) of the wage is based on the assumption that labor market moves instantly and flexibly for the full employment. However, a certain level of unemployment is inevitable due to the rigidity of the wage nature. So, the wage (w) should be modified as

$$w(t) = \frac{\triangle W_0}{L_D(t)}\ e^{\Omega\,(t-t_{L_D}-t_w)} + w_0 \tag{3.144}$$

where t_w is the retardation time of the wage for price change. So, the wage (w) will show the similar movements of Figure 10 just with the delay of $t_{L_D} + t_w$.

So, the unemployment can be rewritten as

$$UE(t) = -\frac{\triangle W_0}{w(t)}\ e^{\Omega\,(t-t_{L_D})}, \tag{3.145}$$

since $L_{D0} = L_{S0}$. Using Eq. (3.144), it will be

$$UE(t) = -\frac{\triangle W_0}{\left(\dfrac{\triangle W_0}{\dfrac{\triangle W_0}{w(t)}\ e^{\Omega\,(t-t_{L_D})} + L_{D0}}\ e^{\Omega\,(t-t_{L_D}-t_w)} + w_0\right)}\ e^{\Omega\,(t-t_{L_D})} \tag{3.145}$$

$$\approx -\frac{\triangle W_0}{w_0} e^{\Omega\,(t-t_{L_D})}\left(1 - \frac{\triangle W_0}{L_{D0}w_0} e^{\Omega\,(t-t_{L_D}-t_w)} + \frac{\triangle {W_0}^2}{{w_0}^2{L_{D0}}^2}\ e^{\Omega\,(2t-2t_{L_D}-t_w)}\right) \tag{3.146}$$

$$\approx -\frac{\triangle W_0}{w_0} e^{\Omega\,(t-t_{L_D})} \tag{3.147}$$

after the elimination of the high order terms. The final equation of unemployment is

$$UE(t) = -\frac{c_{scale2}\beta_{TC}\beta_{pr}}{\left(\alpha_{pr} + \beta_{pr}\right)} \frac{\triangle P_0}{w_0} e^{\Omega\,(t-t_{L_D})} \tag{3.148}$$

since $\triangle W_0 = \frac{c_{scale2}\beta_{TC}\beta_{pr}}{(\alpha_{pr}+\beta_{pr})} \triangle P_0$. For an example, the behavior of the unemployment rate after 50 % price level depreciation is shown in Figure 14.

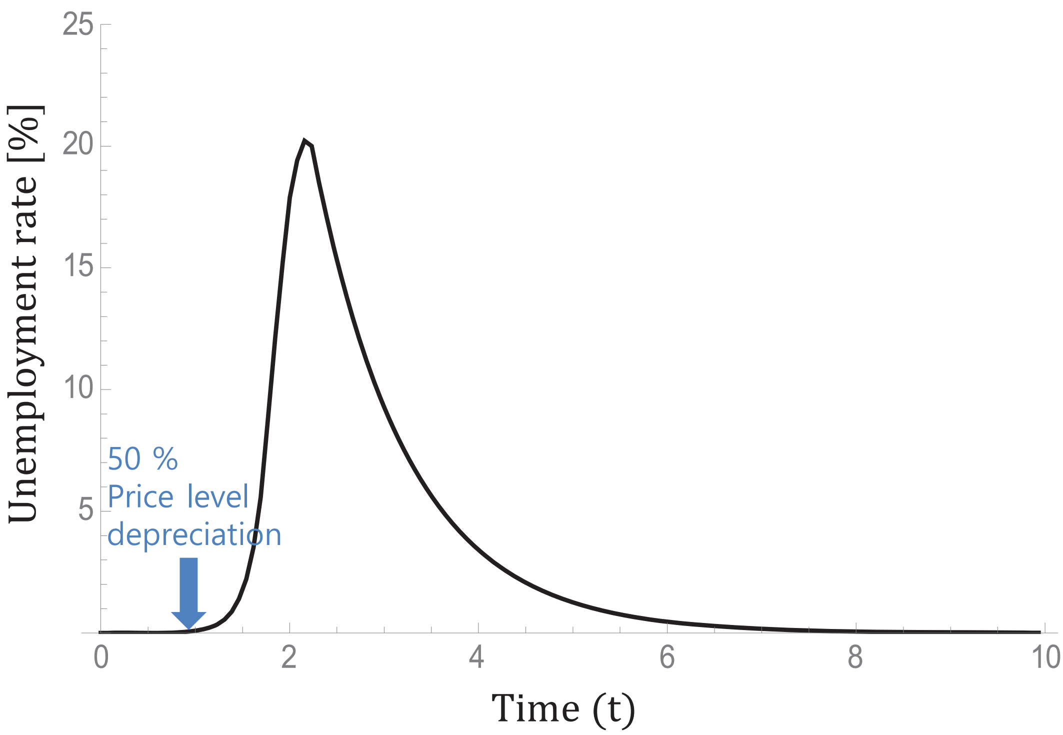


Figure 14. Unemployment rate after 50% price level depreciation when

$\boldsymbol{L_{S0} = 100,\ w_0 = \alpha_{pr} = \beta_{pr} = \beta_{TC} = c_{scale2} = t_{L_D} = 1,\ \Omega = -1,}$

$\boldsymbol{P_{before} = 100}$ and $\boldsymbol{P_{after} = 50}$.

Then, the total income (W) of the house holders from the labor market is

$$W(t) = w(t)L_D(t) = \left(\frac{\triangle W_0}{L_D(t)}\ e^{\Omega\,(t-t_{L_D}-t_w)} + w_0\right) L_D(t) \tag{3.149}$$

$$= \triangle W_0\ e^{\Omega\,(t-t_{L_D}-t_w)} + w_0 L_D(t) \tag{3.150}$$

$$= \triangle W_0\ e^{\Omega\,(t-t_{L_D}-t_w)} + w_0 \frac{\triangle W_0}{w(t)}\ e^{\Omega\,(t-t_{L_D})} + w_0 L_{D0} \tag{3.151}$$

$$\approx \triangle W_0\ \frac{e^{\Omega\,(t-t_{L_D}-t_w)} + \ e^{\Omega\,(t-t_{L_D})}}{2} + W_0 \tag{3.152}$$

where $W_0 = w_0 L_{D0} = \frac{TC_0 \beta_{pr}\, c_{scale2}}{\left(\alpha_{pr} + \beta_{pr}\right)}$ from Eq. (3.139) and Eq. (3.142). Eq. (3.152) will be

$$W(t) = \frac{\triangle W_0}{2}\left(\ e^{\Omega\,(t-t_{L_D})} + e^{\Omega\,(t-t_{L_D}-t_w)}\right) + W_0 \tag{3.153}$$

where $\triangle W_0 = \frac{\beta_{pr}\, c_{scale2}}{\alpha_{pr} + \beta_{pr}} \beta_{TC} \triangle P_0$ and $W_0 = \frac{\beta_{pr}\, c_{scale2}}{\alpha_{pr} + \beta_{pr}} TC_0$. For example, the total income changes of the house holders after 50 % price drop are shown in Figure 15.

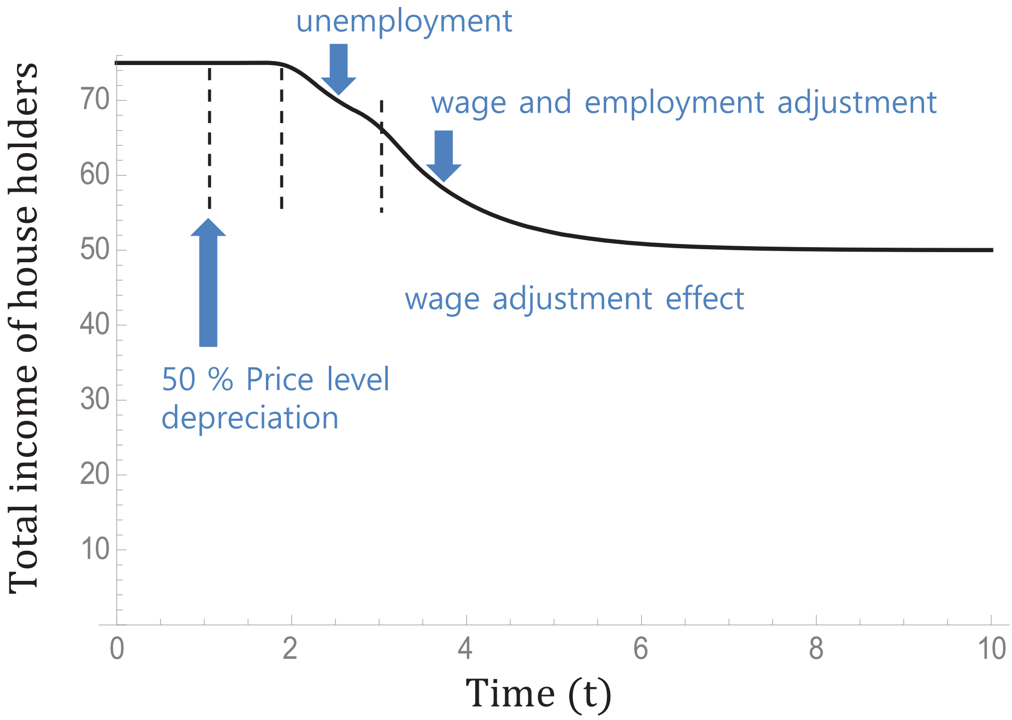


Figure 15. Total income changes of house holders when $\boldsymbol{\alpha_{pr} = \beta_{pr} = c_{scale2} = \beta_{TC} = t_{L_D} = t_w = 1}$, $\boldsymbol{L_{S0} = TC_0 = 100}$, $\boldsymbol{P_{before} = 100}$ and $\boldsymbol{P_{after} = 50}$.

3.3.4 Financial market

As shown in Figure 6, the money inflow into the finance market is

$$M^S = S_H + S_G + S_F + S_f, \tag{3.154}$$

and the money outflow from the finance market will loans to the firm as

$$M^D = wL + K \tag{3.155}$$

from Eq. (3.56). The saving of house holders can be rewritten as

$$S_H(r) = \alpha_{S_H}(r - r_0) + S_{H0} + \alpha_{S_H}^{\Delta r}\frac{dr}{dt} \tag{3.156}$$

from Eq. (3.21), and the money deposit from the government can be expressed as

$$S_G(r) = S_{G0} \tag{3.157}$$

since the government has a right to control the money flow by issuing its treasury bonds. The saving from the firm will be

$$S_F(r) = (1 + r)K_{i-1} + Y_{i-1} - (w_{i-1}L_{i-1} + K_{i-1}) \tag{3.158}$$

where i is the year index, and the saving from foreign countries will be expressed as

$$S_f(r) = ex_{\$}\frac{r - r^B}{\frac{ex_{\$}}{a_r} + \frac{ex_{\$}^B}{b_r}} \tag{3.158}$$

where ex is an exchange rate, $a_r = \alpha_r^A + \beta_r^A$ and $b_r = \alpha_r^B + \beta_r^B$ from Eq. (3.81). So, the net money demand can be

$$ND_M \equiv M^D - M^S = wL + K - \left(S_H + S_G + S_F + S_f\right) \quad (3.159)$$

$$= 2wL + (1-r)K - \alpha_{S_H}(r - r_0) - S_{H0} - S_{G0} - Y - ex_\$ \frac{r - r^B}{\frac{ex_\$}{a_r} + \frac{ex_\$^B}{b_r}} - \alpha_{S_H}^{\Delta r} \frac{dr}{dt} \quad (3.160)$$

when $K_{i-1} \approx K_i$, $w_{i-1} \approx w_i$, $L_{i-1} \approx L_i$, $Y_{i-1} \approx Y_i$. Using Eq. (3.57), Eq. (3.160) can be rewritten as

$$ND_M \approx \left(\frac{1}{r} - 1\right) \frac{\alpha_{pr} c_{scale2}}{\alpha_{pr} + \beta_{pr}} \left(\beta_{TC}(P - P_0) + TC_0\right) - \alpha_{S_H}(r - r_0) - ex_\$ \frac{r - r^B}{\frac{ex_\$}{a_r} + \frac{ex_\$^B}{b_r}} + 2W_0 - S_{H0} - S_{G0} - Y_0 - \alpha_{S_H}^{\Delta r} \frac{dr}{dt} \quad (3.161)$$

where $Y_0 = \beta_S \left(\frac{c_b}{c_a} + \frac{c_c}{c_b}\right)^2 + Q_s|_{P=0} \left(\frac{c_b}{c_a} + \frac{c_c}{c_b}\right)$ from Eq. (3.135), $W_0 = \frac{\beta_{pr}\, c_{scale2}}{\alpha_{pr} + \beta_{pr}} TC_0$ from Eq. (3.147). So, the net demand (ND_M) of money increases as price (P) goes up and as interest rate (r) goes down. Under the equilibrium condition ($ND_M = 0$), Eq. (3.161) will be

$$\frac{\alpha_{pr} c_{scale2}}{\alpha_{pr} + \beta_{pr}} \left(\beta_{TC}(P - P_0) + TC_0\right) = -\alpha_{S_H} \frac{r(r - r_0)}{r - 1} - \frac{ex_\$}{\frac{ex_\$}{a_r} + \frac{ex_\$^B}{b_r}} \frac{r(r - r^B)}{r - 1} - \frac{r}{r - 1}\left(-2W_0 + S_{H0} + S_{G0} + Y_0 + \alpha_{S_H}^{\Delta r} \frac{dr}{dt}\right), \quad (3.162)$$

The price level (P) and the interest rate (r) have a negative proportional relation as shown in Figure 16.

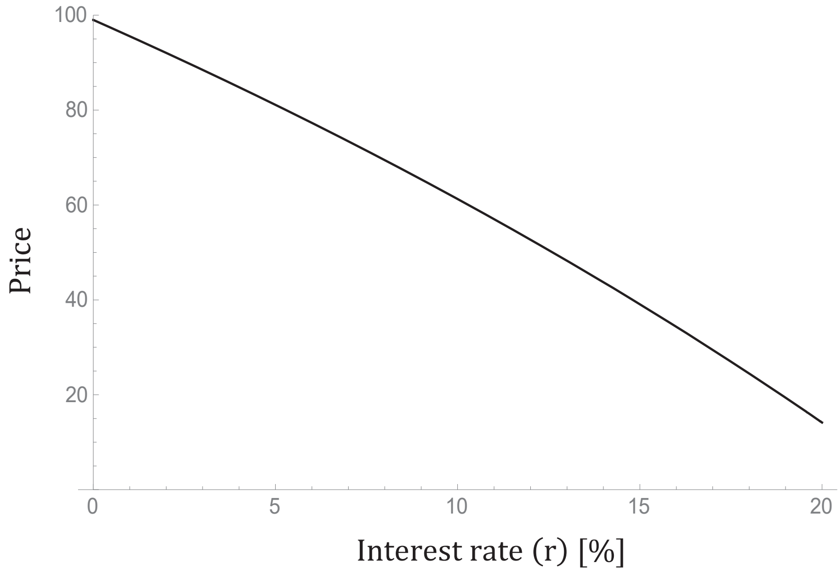


Figure 16. Price changes of the commodity market with the interest rate changes when $\alpha_{pr} = \beta_{pr} = c_{scale2} = \beta_{TC} = TC_0 = \alpha_{S_H} = ex_{\$} = ex_{\$}^{B} = 1, r_0 = 0.1,\ r^B = 0.01, W_0 = P_0 = 100, S_{H0} = 10, S_{G0} = 10, Y_0 = 10, a_r = b_r = 2, \alpha_{S_H}^{\Delta r} = 0$.

3.3.5 Exchange Market

Let us move on to the exchange market. The balance of payments (Bp) for the foreign money will be written as

$$Bp = NX + S_f \tag{3.163}$$

from Eq. (3.4) as shown in Figure 6. The foreign saving can be rewritten as

$$S_f(r) = ex_\$ \frac{r - r^f}{\frac{ex_\$}{a_r} + \frac{ex_\$^f}{a_r^f}} + \alpha_f^{\Delta r} \frac{d\,r}{dt} \tag{3.164}$$

with its exchange rate ($ex_\$$) and the exchange rate ($ex_\$^f$) of a foreign country from Eq. (3.158). The net exports can be also rewritten as

$$NX = P\,Q_{trade} = \frac{ex_\$}{ex_\$^B}\,P \frac{ex_\$ P^{foreign} - ex_\$^f P}{\frac{ex_\$}{a} + \frac{ex_\$^f}{a^f}} \tag{3.165}$$

with the exchange rates ($ex_\$,\ ex_\$^B$). Then the balance of payments will be

$$Bp = \frac{ex_\$}{ex_\$^B}\,P \frac{ex_\$ P^f - ex_\$^f P}{\frac{ex_\$}{a} + \frac{ex_\$^f}{a^f}} + ex_\$ \frac{r - r^f}{\frac{ex_\$}{a_r} + \frac{ex_\$^f}{a_r^f}} + \alpha_f^{\Delta r} \frac{d\,r}{dt} \tag{3.166}$$

With Eq. (3.164) and Eq. (3.165). Eq.(3.166) will be

$$\left(Bp - \alpha_f^{\Delta r}\frac{d\,r}{dt}\right)\left(\frac{ex_\$}{a_r} + \frac{ex_\$^f}{a_r^f}\right)\left(\frac{ex_\$}{a} + \frac{ex_\$^f}{a^f}\right)$$
$$= ex_\$\ P(ex_\$ P_\$^f - P)(\frac{ex_\$}{a_r} + \frac{ex_\$^f}{a_r^f}) + (r - r^B) ex_\$(\frac{ex_\$}{a} + \frac{ex_\$^f}{a^f}) \tag{3.167}$$

or

$$P^2 - ex_{\$}P_{\$}^{f}P - \left(\frac{r - r^f}{\frac{ex_{\$}}{a_r} + \frac{ex_{\$}^{f}}{a_r^{f}}} - \frac{Bp - \alpha_f^{\Delta r}\frac{d\,r}{dt}}{ex_{\$}}\right)\left(\frac{ex_{\$}}{a} + \frac{ex_{\$}^{f}}{a^{f}}\right) = 0 \tag{3.168}$$

where $P_{\$}^{f} = \frac{P^f}{ex_{\$}^{f}}$. When $Bp = 0$ and $\alpha_f^{\Delta r} = 0$, the price level can be

$$P = \frac{ex_{\$}P_{\$}^{f}}{2} \pm \sqrt{\left(\frac{ex_{\$}P_{\$}^{f}}{2}\right)^2 + (r - r^{foreign})\left(\frac{\frac{ex_{\$}}{a} + \frac{ex_{\$}^{f}}{a^{f}}}{\frac{ex_{\$}}{a_r} + \frac{ex_{\$}^{f}}{a_r^{f}}}\right)} \tag{3.169}$$

or

$$P \approx \frac{ex_{\$}}{ex_{\$}^{f}}P^f + \frac{r - r^f}{\frac{ex_{\$}}{ex_{\$}^{f}}P^f}\left(\frac{\frac{ex_{\$}}{a} + \frac{ex_{\$}^{f}}{a^{f}}}{\frac{ex_{\$}}{a_r} + \frac{ex_{\$}^{f}}{a_r^{f}}}\right) \tag{3.170}$$

from Eq. (3.168). The exchange rate ($ex_{\$}$) from Eq. (3.170) can be expressed as

$$ex_{\$} \approx \frac{P}{2\,P_{\$}^{f}} + \frac{1}{2}\sqrt{\left(\frac{P}{P_{\$}^{f}}\right)^2 - 4\frac{r - r^f}{{P_{\$}^{f}}^2}\left(\frac{\frac{ex_{\$0}}{a} + \frac{ex_{\$}^{f}}{a^{f}}}{\frac{ex_{\$0}}{a_r} + \frac{ex_{\$}^{f}}{a_r^{f}}}\right)} \tag{3.171}$$

or

$$ex_{\$} \approx \frac{P}{\frac{P^f}{ex_{\$}^{f}}} - \frac{r - r^f}{P\,\frac{P^f}{ex_{\$}^{f}}}\left(\frac{\frac{ex_{\$0}}{a} + \frac{ex_{\$}^{f}}{a^{f}}}{\frac{ex_{\$0}}{a_r} + \frac{ex_{\$}^{f}}{a_r^{f}}}\right) \tag{3.172}$$

under the assumption that exchange rates ($ex_\$$ and $ex_\$^f$) for the scaling a and a^f are fixed. The first term of Eq. (3.170) tells the law of one price, and the second term shows the contribution parameter induced by the interest rate difference to the law of one price. The exchange rate changes for price changes are plotted in Figure 17.

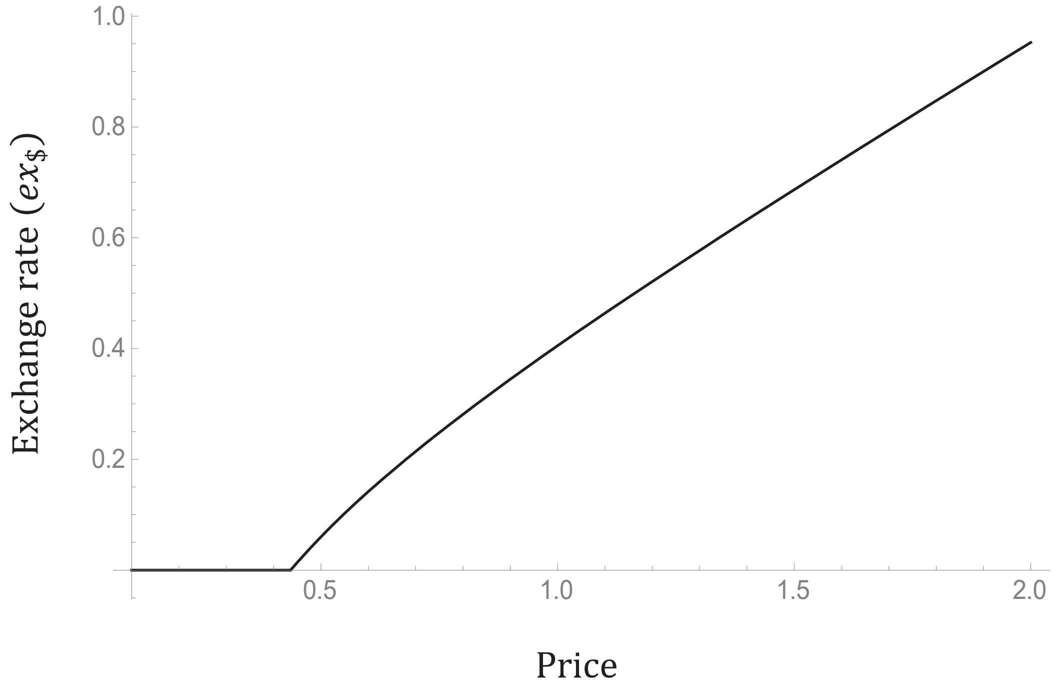


Figure 17. Exchange rate changes ($ex_\$$) for price level changes (P) for when $ex_\$^f = ex_{\$0} = 1$, r = 0.2, $r^f = 0.01$, $P^f = a^f = a = a_r = a_r^f = 2$,

3.4 Trade

In international trade, the comparative advantage of Ricardian model [63] and the factor proportions of Hecksher-Ohlin model [64] are widely used as the main theory of the international trades. The momentum of global value chains is often explained by Ricardian model. In my point of view, those theories are the consequence of the successful trade, and the trade itself is just an action of merging markets in some degrees. The actions can be successful or unsuccessful in enhancing economy efficiency of the markets, and the actions may be still tried somewhere now. Here, a mathematical approach will be taken to figure out the core characteristics of trade.

Does trade always make the market value bigger?

Let us start the same situation with country A and country B in the previous question. After full trade or market merging, the total demand (D^{Tot}) and the total supply (S^{Tot}) can be written as

$$D^{Tot}(p) = D^A(p) + D^B(p) = -\alpha^A\,(p - p_0^A) - \alpha^B\,(p - p_0^B) + Q_0^A + Q_0^B \qquad (3.173)$$

$$S^{Tot}(p) = S^A(p) + S^B(p) = \beta^A\,(p - p_0^A) + \beta^B\,(p - p_0^B) + Q_0^A + Q_0^B \qquad (3.174)$$

using equations from Eq. (3.66) to Eq. (3.69). And the equilibrium price (p) and market quantity (Q) after full trade (complete market merging) can be derived as

$$p_0^{Tot} = \frac{(\alpha^A + \beta^A)p_0^A + (\alpha^B + \beta^B)p_0^B}{(\alpha^A + \beta^A) + (\alpha^B + \beta^B)} \qquad (3.175)$$

using Eq. (3.173) and Eq. (3.174), and

$$Q_0^{Tot} = \frac{\beta^A\beta^B\left(\frac{\alpha^A}{\beta^A} - \frac{\alpha^B}{\beta^B}\right)}{(\alpha^A + \beta^A) + (\alpha^B + \beta^B)}(p_0^A - p_0^B) + Q_0^A + Q_0^B \tag{3.176}$$

$$\Delta Q_0 = \frac{\beta^A\beta^B\left(\frac{\alpha^A}{\beta^A} - \frac{\alpha^B}{\beta^B}\right)}{(\alpha^A + \beta^A) + (\alpha^B + \beta^B)}(p_0^A - p_0^B) \tag{3.177}$$

using Eq. (3.174) and Eq. (3.175). Eq. (3.177) says that the total market quantity changes with the difference of the price elasticity ratio (α/β) since $(p_0^A - p_0^B) > 0$. Without any difference of the price elasticity ratio (α/β), there is no change in the market quantity (Q).

Now, the market value (p*Q) after the full trade can be

$$\begin{aligned} MV &= p_0^{Tot} Q_0^{Tot} \\ &= \frac{(p_0^A - p_0^B)}{(a+b)}\left(\beta^A\beta^B \frac{(a\, p_0^A + b\, p_0^B)}{(a+b)}\left(\frac{\alpha^A}{\beta^A} - \frac{\alpha^B}{\beta^B}\right) + (a\, Q_0^B - b\, Q_0^A)\right) \\ &\quad + p_0^A Q_0^A + p_0^B\, Q_0^B \end{aligned} \tag{3.178}$$

where MV refers market value, $a = \alpha^A + \beta^A$ and $\mathrm{b} = \alpha^B + \beta^B$ using Eq. (3.175) and Eq. (3.176). The change of the market value after the full trade can be expressed as

$$\begin{aligned} \Delta MV &= p_0^{Tot} Q_0^{Tot} - (\, p_0^A Q_0^A + p_0^B\, Q_0^B) \\ &= \frac{(p_0^A - p_0^B)}{(a+b)}\left(\beta^A\beta^B \frac{(a\, p_0^A + b\, p_0^B)}{(a+b)}\left(\frac{\alpha^A}{\beta^A} - \frac{\alpha^B}{\beta^B}\right) + a\, b(\frac{Q_0^B}{b} - \frac{Q_0^A}{a})\right) \end{aligned} \tag{3.179}$$

from Eq. (3.178). So, market value can be increased or decreased with the difference of the price elasticity ratio (α/β) and the difference of the effective market size (Q_0^A/a or Q_0^B/b). In ordinary cases, the change of market value (ΔMV) is positive since the market size (Q_0^A) of a country with higher price (p_0^A) is usually smaller than that of the other country with lower price (p_0^B). However, when a small country has lower price with any reason such as technology innovation than a big country, the total market wealth of the commodity can be shrunk after trade though the consumer's surplus gets bigger.

What is the effect of trade constraints on market value?

Trade intrinsically needs cost such as transport fee, and some policies such as tariff or quota are used to protect their industries. Under the circumstance, the constraints can be easily reflected on the previous works with small adjustments. In the import or export quota policy, the price after trade can be

$$p_{0\,after\,trade}^{A} = p_0^A - \frac{1}{a} Q_{quota} \tag{3.180}$$

$$p_{0\,after\,trade}^{B} = p_0^B + \frac{1}{b} Q_{quota} \tag{3.181}$$

$$Q_{0\,after\,trade}^{A} = Q_0^A + \frac{\alpha^A}{a} Q_{quota} \tag{3.182}$$

$$Q_{0\,after\,trade}^{B} = Q_0^B - \frac{\alpha^B}{b} Q_{quota} \tag{3.183}$$

from Eq. (3.74), Eq. (3.75), Eq. (3.66) and Eq. (3.68). The change of the market value will be

$$\Delta MV = p_{0\,after\,trade}^{A} * Q_{0\,after\,trade}^{A} + p_{0\,after\,trade}^{B} * Q_{0\,after\,trade}^{B} - p_0^A * Q_0^A - p_0^B * Q_0^B \tag{3.184}$$

$$= \left(p_0^A - \frac{Q_{quota}}{a}\right)\left(Q_0^A + \frac{\alpha^A Q_{quota}}{a}\right) + \left(p_0^B + \frac{Q_{quota}}{b}\right)\left(Q_0^B - \frac{\alpha^B Q_{quota}}{b}\right) - p_0^A Q_0^A - p_0^B Q_0^B \tag{3.185}$$

$$= \left(\left(\frac{\alpha^A p_0^A}{a} - \frac{\alpha^B p_0^B}{b}\right) - \left(\frac{\alpha^A}{a^2} + \frac{\alpha^B}{b^2}\right) Q_{quota} + \left(\frac{Q_0^B}{b} - \frac{Q_0^A}{a}\right)\right) Q_{quota} \tag{3.186}$$

As the quota gets the full trade quantity ($Q_{quota} = Q_{trade} = \frac{p_0^A - p_0^B}{a+b} ab$), the change of the market value of Eq. (3.186) is approaching Eq. (3.179). So, the quota policy means that partial merging of two markets, and the price adjustment can be taken partially.

If there is no trade volume restriction, and if the exchange rate and the trade cost including tariff or subsidy need to be considered, the trade quantity and the price can be

$$Q_{trade} = \frac{e^A p_0^A - (e^B p_0^B + cost)}{\frac{1}{\alpha^A + \beta^A} + \frac{1}{\alpha^B + \beta^B}} \tag{3.187}$$

$$p_0^{after\ trade} = \frac{(\alpha^A + \beta^A) e^A p_0^A + (\alpha^B + \beta^B)(e^B p_0^B + cost)}{(\alpha^A + \beta^A) + (\alpha^B + \beta^B)} \tag{3.188}$$

where e and cost are respectively the exchange rate to key currency and trade cost including transport fee, tariff or subsidy (negative tariff) from Eq. (3.76) and Eq. (3.75). The change of the market value after trade with the constraint will be

$$\Delta MV = \frac{\left(e^A p_0^A - (e^B p_0^B + cost)\right)}{(a+b)} * \left(\beta^A \beta^B \frac{\left(a e^A p_0^A + b(e^B p_0^B + cost)\right)}{(a+b)} \left(\frac{\alpha^A}{\beta^A} - \frac{\alpha^B}{\beta^B} \right) + a\, b \left(\frac{Q_0^B}{b} - \frac{Q_0^A}{a} \right) \right), \tag{3.189}$$

and the loss of market value can be

$$\text{loss} = MV_{full\ trade} - (MV_{constrainted\ trade} + cost * Q_{constrainted\ trade}) \tag{3.190}$$

$$= p_0^{full\ trade} Q_0^{full\ trade} - \left(p_0^{full\ trade} + p_{cost}\right)\left(Q_0^{full\ trade} - Q_{cost}\right) - cost\left(Q_0^{full\ trade} - Q_{cost}\right) \tag{3.191}$$

where

$$p_0^{full\ trade} = \frac{(a\, e^A\, p_0^A + b\, e^B\, p_0^B)}{(a+b)} \tag{3.192}$$

$$p_{cost} = \frac{b\ cost}{(a+b)} \tag{3.193}$$

$$Q_0^{full\ trade} = \frac{e^A p_0^A - e^B p_0^B}{\frac{1}{a} + \frac{1}{b}} \tag{3.194}$$

$$Q_{cost} = \frac{cost}{\frac{1}{a} + \frac{1}{b}} \tag{3.195}$$

and the market value loss can be derived as

$$loss = a\, b\, \frac{(a+2b)\ cost^2 + \left((a+3b)e^B p_0^B - 2be^A\, p_0^A\right) cost}{(a+b)}. \tag{3.196}$$

So, the loss can be induced by the quadratic function of cost. The most efficient way to promote the trade can be the policy reducing the trade cost. After 1990s, world trade volume is exponentially increased and its main cause has been referred to be the developments in information technology (IT) and transports. Eq. (3.196) gives a theoretical explanation about the connection between the development in IT and transports and trade volume.

What makes trade patterns?

Case 1: one importing country and two exporting countries

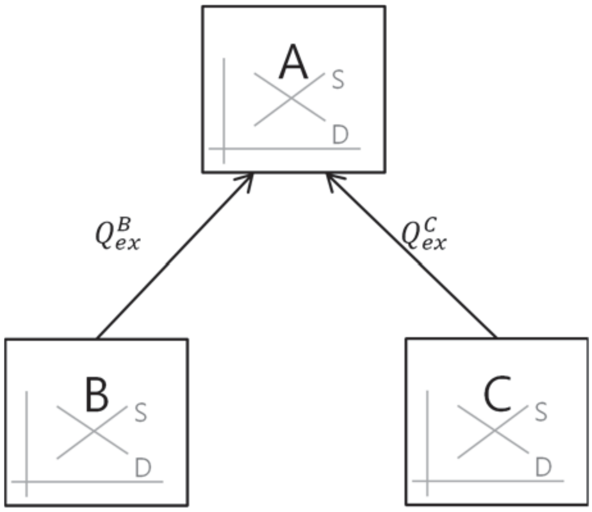


Figure 18. 2 countries (country B and country C) exports commodity to a single country (country A)

Let us consider the trade among three countries (country A, country B, country C); one country (country A) is importing, and the other two countries (country B and country C) are exporting to country A as shown in Figure 18. Two exporting countries are assumed to have different trade cost ($cost^B$ and $cost^C$), and the exchange rates are assumed to be fixed and identical. After the trade, each country has its own equilibrium price (p_{trade}^{A}, p_{trade}^{B} and p_{trade}^{C}), and their relations can be expressed as

$$p_{trade}^{A} = p_{trade}^{B} + cost^{B} = p_{trade}^{C} + cost^{C} \tag{3.197}$$

, and

$$p_{trade}^{A} = p_0^A - \frac{Q_{im}^{A}}{a} = p_0^A - \frac{Q_{ex}^{B} + Q_{ex}^{C}}{a} \quad (3.198)$$

$$p_{trade}^{B} = p_0^B + \frac{Q_{ex}^{B}}{b} \quad (3.199)$$

$$p_{trade}^{C} = p_0^C + \frac{Q_{ex}^{C}}{c} \quad (3.200)$$

where $a = \alpha^A + \beta^A$, $\mathrm{b} = \alpha^B + \beta^B$ and $\mathrm{c} = \alpha^C + \beta^C$ from Eq. (3.74) and Eq. (3.75). In the trade between country A and country B, the price relation is

$$p_0^A - \frac{Q_{ex}^{B} + Q_{ex}^{C}}{a} = p_0^B + \frac{Q_{ex}^{B}}{b} + cost^B \quad (3.201)$$

$$(\frac{1}{a} + \frac{1}{b})Q_{ex}^{B} + \frac{1}{a}Q_{ex}^{C} = p_0^A - (p_0^B + cost^B) \quad (3.202)$$

using the trade between country A and country C, the price relation can be

$$p_0^A - \frac{Q_{ex}^{B} + Q_{ex}^{C}}{a} = p_0^C + \frac{Q_{ex}^{C}}{c} + cost^C \quad (3.203)$$

$$\frac{1}{a}Q_{ex}^{B} + \left(\frac{1}{a} + \frac{1}{c}\right)Q_{ex}^{C} = p_0^A - (p_0^C + cost^C). \quad (3.204)$$

Then, from Eq. (3.202) and Eq. (3.204),

$$Q_{ex}^{B} = \frac{\left(\frac{1}{a} + \frac{1}{c}\right)\Delta p_0^B - \frac{1}{a}\Delta p_0^C}{\left(\frac{1}{a} + \frac{1}{b}\right)\left(\frac{1}{a} + \frac{1}{c}\right) - \left(\frac{1}{a}\right)^2} \quad (3.205)$$

$$Q_{ex}^{B} = (ab)\frac{(a + c)\Delta p_0^B - c\Delta p_0^C}{(a + b)(a + c) - bc} \quad (3.206)$$

$$Q_{ex}^{C} = \frac{\left(\frac{1}{a}+\frac{1}{b}\right)\Delta p_0^C - \frac{1}{a}\Delta p_0^B}{\left(\frac{1}{a}+\frac{1}{b}\right)\left(\frac{1}{a}+\frac{1}{c}\right) - \left(\frac{1}{a}\right)^2} \tag{3.207}$$

$$Q_{ex}^{C} = (ac)\frac{(a+b)\Delta p_0^C - b\Delta p_0^B}{(a+b)(a+c) - bc} \tag{3.208}$$

where $\Delta p_0^B = p_0^A - (p_0^B + cost^B)$, $\Delta p_0^C = p_0^A - (p_0^C + cost^C)$, $a = \alpha^A + \beta^A$, $\mathrm{b} = \alpha^B + \beta^B$ and $\mathrm{c} = \alpha^C + \beta^C$. The price after the trade in country A is

$$p_{trade}^{A} = p_0^A - \frac{a(\mathrm{b}\,\Delta p_0^B + c\,\Delta p_0^C)}{\left((a+b)(a+c) - (bc)\right)} \tag{3.209}$$

from Eq. (3.198), Eq. (3.206) and Eq. (3.208). Now, to examine the trade quantity of country B and country C, the trade quantity ratio (Q_{ex}^B/Q_{ex}^C) is derived as

$$\frac{Q_{ex}^B}{Q_{ex}^C} = \left(\frac{b}{c}\right)\frac{(a+c)\Delta p_0^B - c\Delta p_0^C}{(a+b)\Delta p_0^C - b\Delta p_0^B} \tag{3.210}$$

from Eq. (3.206) and Eq. (3.208).

When two countries (country B, country C) have the same difference $(\Delta p_0^C = \Delta p_0^B)$ in trade with the exported country (country A), the trade quantity ratio is

$$\frac{Q_{ex}^B}{Q_{ex}^C} = \frac{b}{c} \tag{3.211}$$

, and the trade pattern (Q_{ex}^B/Q_{ex}^C) is formed according to the price elasticity ratio (b/c). The price elasticity (a, b and c) is the slope of net demand ($N(p) = D(p) - S(p)$) of a country, and it also presents the market size of the country since the quantity change of a big market is much bigger than that of a small market when the price of a commodity changes identically. From Eq. (3.211), the allocation of the trade quantity of the importing country is exactly proportional to the market size of the exporting country. This gives the theoretical explanation of "Gravity theory" in trade.

Now, let us consider the case that the two countries have the identical price elasticity ($b = c$) of the net demand. Then, Eq. (3.210) will be

$$\frac{Q_{ex}^B}{Q_{ex}^C} = \frac{\left(1 + \frac{a}{b}\right)\frac{\Delta p_0^B}{\Delta p_0^C} - 1}{\left(1 + \frac{a}{b}\right) - \frac{\Delta p_0^B}{\Delta p_0^C}} \tag{3.212}$$

, and it says the price gap of each country determines the trade pattern, and the pattern will be more firmly constructed as the market size of the importing country (country A) gets bigger. Eq. (3.210) gives a clue to understand the current international trade pattern. Market size and t the price level of a commodity compared with those of the competing countries is not only key factor of the trade but also the market size of the commodity in the exporting country plays a role to determine the trade quantity. In trade study, "Gravity theory" is the representative description about the trade pattern, and those are mainly based on the trade data. Eq. (3.210) strongly supports "Gravity theory" and it can also explain other patterns of the trade networks

Case 2: two importing countries and two exporting countries

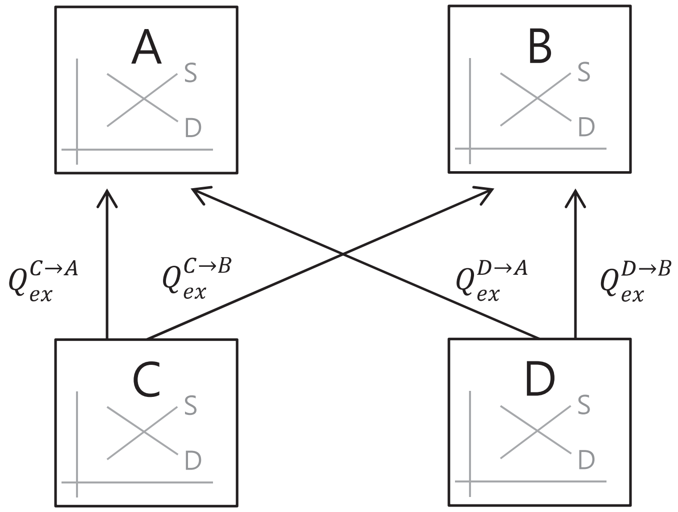


Figure 19. 2 countries (country C and country D) exports commodity to two countries (country A and country B)

Let us consider trade among four countries: two countries (country A and country B) are just importing from the other two exporting countries (country C and country D) as shown in Figure 19. After trade, the prices of a commodity of the countries are

$$p_{trade}^{A} = p_0^A - \frac{Q_{im}^{A}}{a} = p_0^A - \frac{Q_{ex}^{C\to A} + Q_{ex}^{D\to A}}{a} \tag{3.213}$$

$$p_{trade}^{B} = p_0^B - \frac{Q_{im}^{B}}{b} = p_0^B - \frac{Q_{ex}^{C\to B} + Q_{ex}^{D\to B}}{b} \tag{3.214}$$

$$p_{trade}^{C} = p_0^C + \frac{Q_{ex}^{C}}{c} = p_0^C + \frac{Q_{ex}^{C\to A} + Q_{ex}^{C\to B}}{c} \tag{3.215}$$

$$p_{trade}^{D} = p_0^D + \frac{Q_{ex}^{D}}{d} = p_0^D + \frac{Q_{ex}^{D\to A} + Q_{ex}^{D\to B}}{d} \tag{3.216}$$

where $a = \alpha^A + \beta^A$, $\mathrm{b} = \alpha^B + \beta^B$, $c = \alpha^C + \beta^D$, $d = \alpha^D + \beta^D$, and Q_{im} and Q_{ex} are respectively import quantity of a country and export quantity of a country, and superscript refers the importing country or its exporting origin and destination. The prices should be the relation as

$$p_{trade}^{A} = p_{trade}^{C} + cost^{C\to A} = p_{trade}^{D} + cost^{D\to A} \tag{3.217}$$

$$p_{trade}^{B} = p_{trade}^{C} + cost^{C\to B} = p_{trade}^{D} + cost^{D\to B} \tag{3.218}$$

From Eq. (3.197). So, the price relations can be expressed as

$$p_0^A - \frac{Q_{ex}^{C\to A} + Q_{ex}^{D\to A}}{a} = p_0^C + \frac{Q_{ex}^{C\to A} + Q_{ex}^{C\to B}}{c} + cost^{C\to A} \tag{3.219}$$

$$p_0^A - \frac{Q_{ex}^{C\to A} + Q_{ex}^{D\to A}}{a} = p_0^D + \frac{Q_{ex}^{D\to A} + Q_{ex}^{D\to B}}{d} + cost^{D\to A} \tag{3.220}$$

$$p_0^B - \frac{Q_{ex}^{C\to B} + Q_{ex}^{D\to B}}{b} = p_0^C + \frac{Q_{ex}^{C\to A} + Q_{ex}^{C\to B}}{c} + cost^{C\to B} \tag{3.221}$$

$$p_0^B - \frac{Q_{ex}^{C\to B} + Q_{ex}^{D\to B}}{b} = p_0^D + \frac{Q_{ex}^{D\to A} + Q_{ex}^{D\to B}}{d} + cost^{D\to B} \tag{3.222}$$

from Eq. (3.217) and Eq. (3.218). The trade quantities (export volume) can be derived from

$$\begin{pmatrix} \frac{1}{a}+\frac{1}{c} & \frac{1}{c} & \frac{1}{a} & 0 \\ \frac{1}{c} & \frac{1}{b}+\frac{1}{c} & 0 & \frac{1}{b} \\ \frac{1}{a} & 0 & \frac{1}{a}+\frac{1}{d} & \frac{1}{d} \\ 0 & \frac{1}{b} & \frac{1}{d} & \frac{1}{b}+\frac{1}{d} \end{pmatrix} \begin{pmatrix} Q_{ex}^{C\to A} \\ Q_{ex}^{C\to B} \\ Q_{ex}^{D\to A} \\ Q_{ex}^{D\to B} \end{pmatrix} = \begin{pmatrix} p_0^A - (p_0^C + cost^{C\to A}) \\ p_0^B - (p_0^C + cost^{C\to B}) \\ p_0^A - (p_0^D + cost^{D\to A}) \\ p_0^B - (p_0^D + cost^{D\to B}) \end{pmatrix} \tag{3.223}$$

Let us assume that $= b = c = d = 1$, all cost are 0, $p_0^A = p_0^B$, $p_0^C = p_0^D$ and $p_0^A - p_0^C = \Delta p > 0$. Then

$$Q_{ex}^{C\to A} = Q_{ex}^{D\to B} \tag{3.224}$$

$$Q_{ex}^{C\to B} = Q_{ex}^{D\to A} \tag{3.225}$$

$$Q_{ex}^{C\to A} + Q_{ex}^{C\to B} = Q_{ex}^{D\to A} + Q_{ex}^{D\to B} = \frac{\Delta p}{2} \tag{3.226}$$

can be derived from Eq. (3.223). If the importing countries are identical and the exporting countries are identical, the trade quantity pattern gets be symmetric for the most efficient trade as shown in Figure 20.

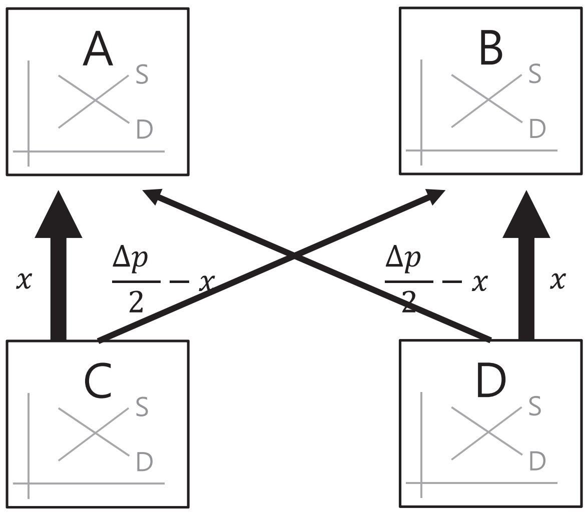


Figure 20. trade pattern of the case 2 with identical importing and exporting conditions

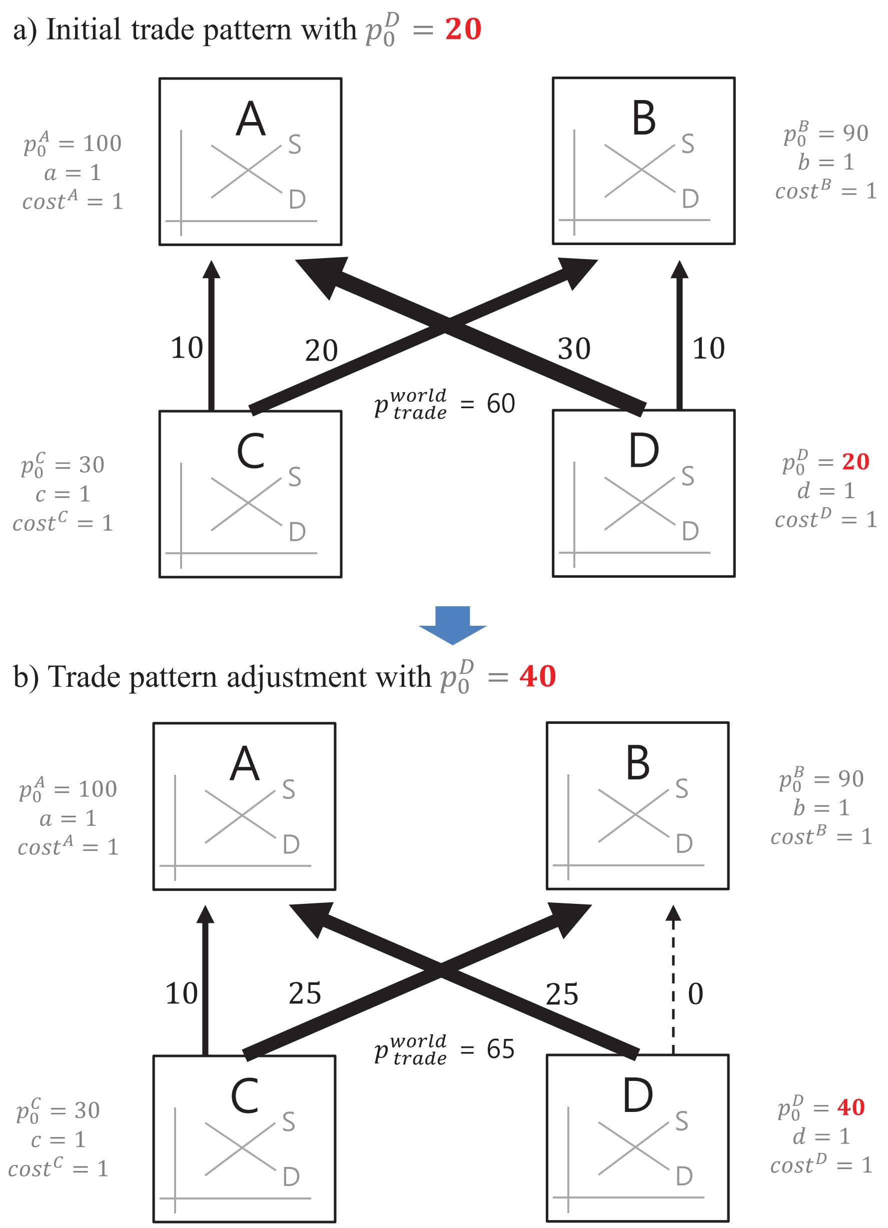


Figure 21. trade pattern adjustment with the price change of Country D.

Now, self-adjustment of the trade network will be examined. Let us assume that $= b = c = d = 1$, all cost are 0, $p_0^A = 100$, $p_0^B = 90$, $p_0^C = 30$ and $p_0^D = 20$. Then Eq. 5-54 gives $Q_{ex}^{C\to B} = 30 - Q_{ex}^{C\to A}$, $Q_{ex}^{D\to A} = 40 - Q_{ex}^{C\to A}$, $Q_{ex}^{D\to B} = Q_{ex}^{C\to A}$, and the world price ($p_{trade}^{world}$)

after trade gets to be 60 from Eq. 5-44. Let p_0^D change to be double (40) to check the impact on the world price with the price changes of the major exporting country, and let others be same as before. Then $Q_{ex}^{C\to B} = 35 - Q_{ex}^{C\to A}$, $Q_{ex}^{D\to A} = 35 - Q_{ex}^{C\to A}$, $Q_{ex}^{D\to B} = Q_{ex}^{C\to A} - 10$. So, the exporting volumes of country C and country D are changed respectively from 30 to 35 and from 40 to 25. The world price (p_{trade}^{world}) after trade gets to be 65. Even when the price of the major exporting country gets to be double, the world price after trade is changed by just 8.3 % through the self-adjustment of the trade network as shown in Figure 21.

Case 3: circular trades among 3 countries

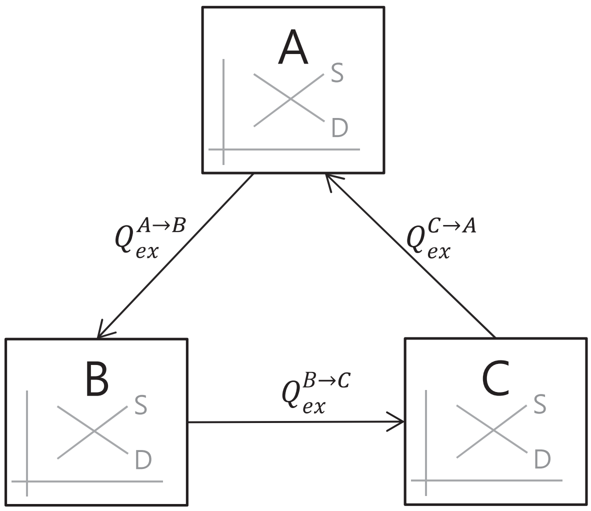


Figure 22. circular trades among 3 countries

Now, let us discuss the trade pattern of the circular trade among 3 countries as shown in Figure 22. After the trade, the prices of three countries will get to be

$$p_{trade}^{A} = p_0^A - \frac{Q_{im}^{A}}{a} = p_0^A - \frac{Q_{ex}^{C\to A} - Q_{ex}^{A\to B}}{a} \tag{3.227}$$

$$p_{trade}^{B} = p_0^B - \frac{Q_{im}^{B}}{b} = p_0^B - \frac{Q_{ex}^{A\to B} - Q_{ex}^{B\to C}}{b} \tag{3.228}$$

$$p_{trade}^{C} = p_0^C - \frac{Q_{ex}^{C}}{c} = p_0^C - \frac{Q_{ex}^{B\to C} - Q_{ex}^{C\to A}}{c} \tag{3.229}$$

where $Q_{ex}^{i\to j} = 0\ or\ Q_{ex}^{j\to i} = 0\ if\ \left|p_0^i - p_0^j\right| \le \left|cost^{i\to j}\right|$ from Eq. (3.74). The prices after the trade can be related as

$$p_{trade}^{A} = p_{trade}^{C} + cost^{C\to A} \tag{3.230}$$

$$p_{trade}^{B} = p_{trade}^{A} + cost^{A\to B} \tag{3.231}$$

$$p_{trade}^{C} = p_{trade}^{B} + cost^{B\to C} \tag{3.232}$$

where $cost^{i\to j} > 0\ if\ p_0^i < p_0^j\ or\ cost^{i\to j} < 0\ if\ p_0^i > p_0^j$ under the assumption that the trade cost is same for forward and backward directions. Eq. (3.231), Eq. (3.232) and Eq. (3.230) can be rewritten as

$$p_0^B - (p_0^A + cost^{A\to B}) = Q_{ex}^{A\to B}\left(\frac{1}{a} + \frac{1}{b}\right) - \frac{Q_{ex}^{B\to C}}{b} - \frac{Q_{ex}^{C\to A}}{a} \tag{3.233}$$

$$p_0^C - (p_0^B + cost^{B\to C}) = -\frac{Q_{ex}^{A\to B}}{b} + Q_{ex}^{B\to C}(\frac{1}{b} + \frac{1}{c}) - \frac{Q_{ex}^{C\to A}}{c} \tag{3.234}$$

$$p_0^A - (p_0^C + cost^{C\to A}) = -\frac{Q_{ex}^{A\to B}}{a} - \frac{Q_{ex}^{B\to C}}{c} + \left(\frac{1}{a} + \frac{1}{c}\right)Q_{ex}^{C\to A}, \tag{3.235}$$

and they can be expressed in a matrix form as

$$\begin{pmatrix} p_0^B - (p_0^A + cost^{A\to B}) \\ p_0^C - (p_0^B + cost^{B\to C}) \\ p_0^A - (p_0^C + cost^{C\to A}) \end{pmatrix} = \begin{pmatrix} \left(\frac{1}{a}+\frac{1}{b}\right) & -\frac{1}{b} & -\frac{1}{a} \\ -\frac{1}{b} & \left(\frac{1}{b}+\frac{1}{c}\right) & -\frac{1}{c} \\ -\frac{1}{a} & -\frac{1}{c} & \left(\frac{1}{c}+\frac{1}{a}\right) \end{pmatrix} \begin{pmatrix} Q_{ex}^{A\to B} \\ Q_{ex}^{B\to C} \\ Q_{ex}^{C\to A} \end{pmatrix}. \quad (3.236)$$

For an example, let us consider the case with $a = b = c = 1$, $cost^{A\to B} = cost^{B\to C} = cost^{C\to A} = 0$, $p_0^A = 30$, $p_0^B = 23$ and $p_0^C = 10$. From Eq. (3.236), the export quantity will be $Q_{ex}^{A\to B} = -x$, $Q_{ex}^{B\to C} = -2 - x$ and $Q_{ex}^{C\to A} = 9 - x$ as Figure 23. So, the actual export quantity of country C is 11, and the imports of country A and country B are respectively 2 and 9. Country B can transfer x quantity from country C to country A.

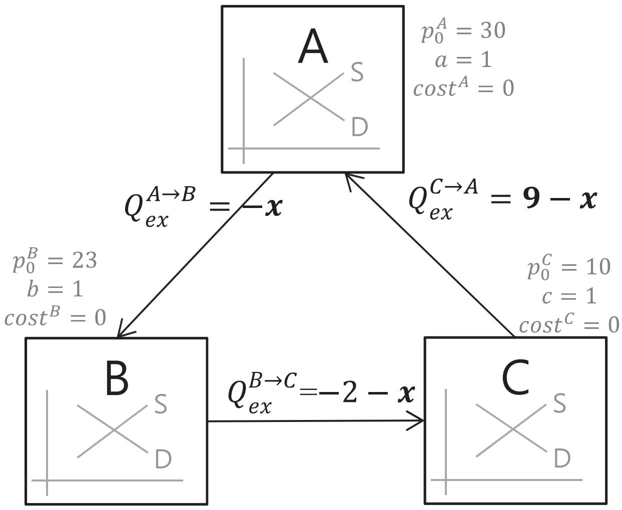


Figure 23. the trade pattern of the case with $\boldsymbol{a = b = c = 1}$, $\boldsymbol{cost^{A\to B} = cost^{B\to C} = cost^{C\to A} = 0}$, $\boldsymbol{p_0^A = 30}$, $\boldsymbol{p_0^B = 20}$ and $\boldsymbol{p_0^C = 10}$

The coefficient matrix of Eq. (3.236) does not have its inverse matrix since

$$\begin{vmatrix} \left(\frac{1}{a}+\frac{1}{b}\right) & -\frac{1}{b} & -\frac{1}{a} \\ -\frac{1}{b} & \left(\frac{1}{b}+\frac{1}{c}\right) & -\frac{1}{c} \\ -\frac{1}{a} & -\frac{1}{c} & \left(\frac{1}{c}+\frac{1}{a}\right) \end{vmatrix} = 0. \tag{3.237}$$

It means that it does not have its equilibrium solution for all cases. The trade equilibrium among several countries cannot be always reached. In other words, endless activities in trade will be taken to resolve the price difference among countries.

Chapter 4. Economy Network Pattern

This chapter presents the figures of the real world economy to understand the patterns of the world economy. The patterns can be used to simulate the behavior of individual countries in the real world and to synthesize networks among countries. So, the world economy can be synthesized by piling or connecting the national economy models in the previous chapter. The data are retrieved from World Bank, International Monetary Fund (IMF), Central Intelligence Agency (CIA) and Organization for Economic Co-operation and Development (OECD). Since the institutes provide only recent data after 1960s or after 1980s, the analysis is focusing on the recent economy. Network analysis method is mainly used to understand the pattern of the world economy. The ultimate concern of macro-economy may be everlasting prosperity or a stable growth. So, the growth rate will be first discussed to figure out the pattern of the world economy. The growth is supported by trade and finance as discussed in Chapter 3. So, this chapter consists of growth rate analysis in Section 4.1, trade pattern analysis in Section 4.2 and finance pattern analysis in Section 4.3.

4.1 Growth Rate Pattern

In this section, growth rate will be discussed not growth since economy is a dynamically moving thing. Its dynamic changes can be more properly reflected on the growth rate. The main goal of this section is figuring out the research questions: "Who are the core countries, and who are the periphery countries?" and "How are they connected?". First, the growth rate will be discussed by the classifications of World Bank. And the correlations of the core countries will be examined in growth rate domain. Based on the correlations of the growth rates, global network of the countries will be discussed.

4.1.1. Income group and regional contributions

The dependency of the world growth rates is examined by income groups in this section. World Bank classifies the countries into 4 incomes: high (79 countries), upper middle (60 countries), lower middle (47 countries) and low income (31 countries). The changes of their GDP growth rates are plotted in Figure 24 a). Their contributions to the world growth rates are examined, and the contribution portions are calculated by the sampling span as shown in Figure 24 b). Since the error abruptly grows up over 12-year span, the spans under 12 years are considered in the calculation of the contributions to the world growth rates, and the contributions of the income groups are turned out to be 70.4 %, 23.5 %, 7.7 % and 0.5 % respectively for high, upper middle, lower middle and low income groups with statistical checks as shown in Figure 24 c). It seems to be quite surprising that more than 70 % of the world growth rate is determined by the high income group. The value tells us the high income group has played monopolistic roles in the world economy. Adding that of the upper middle income group on the contribution of the high income, the total contribution goes to 93.9 %. In a short word, the world economy seems to be for the high income group and the upper middle income group. However, the number of countries in the high or upper middle income groups is 139, representing 64 % of the whole countries. The contributions seem to have no big change for last 30 years, but the contribution of the high income group tends to be slightly decreasing while that of the upper middle group is slightly increasing after 2009 as shown in Figure 25. The contribution of the high income group changes from 74.9 % in 2007 to 65.9 % in 2018, while that of the upper middle income group does from 17.17 % in 2007 to 25.6 % in 2018. However total portions of the two groups are the same to be 91.5 %. If the trends go on, the governance of the world economy could be thought to be changing from a monopoly of the high income group to collaborations between high income group and upper middle income group. From another scope, these trends can be regarded as decoupling after global synchronization of the business cycles.

The contribution analysis of regional contributions is shown in Figure 26. East Asia & Pacific, Europe & Central Asia and North America are ranked at top 3 among 7 regional classifications respectively with 34.6 %, 35.7 % and 22.1 %. The total contribution of this top 3 is 92.4 %. The world growth seems to be determined by these regions.

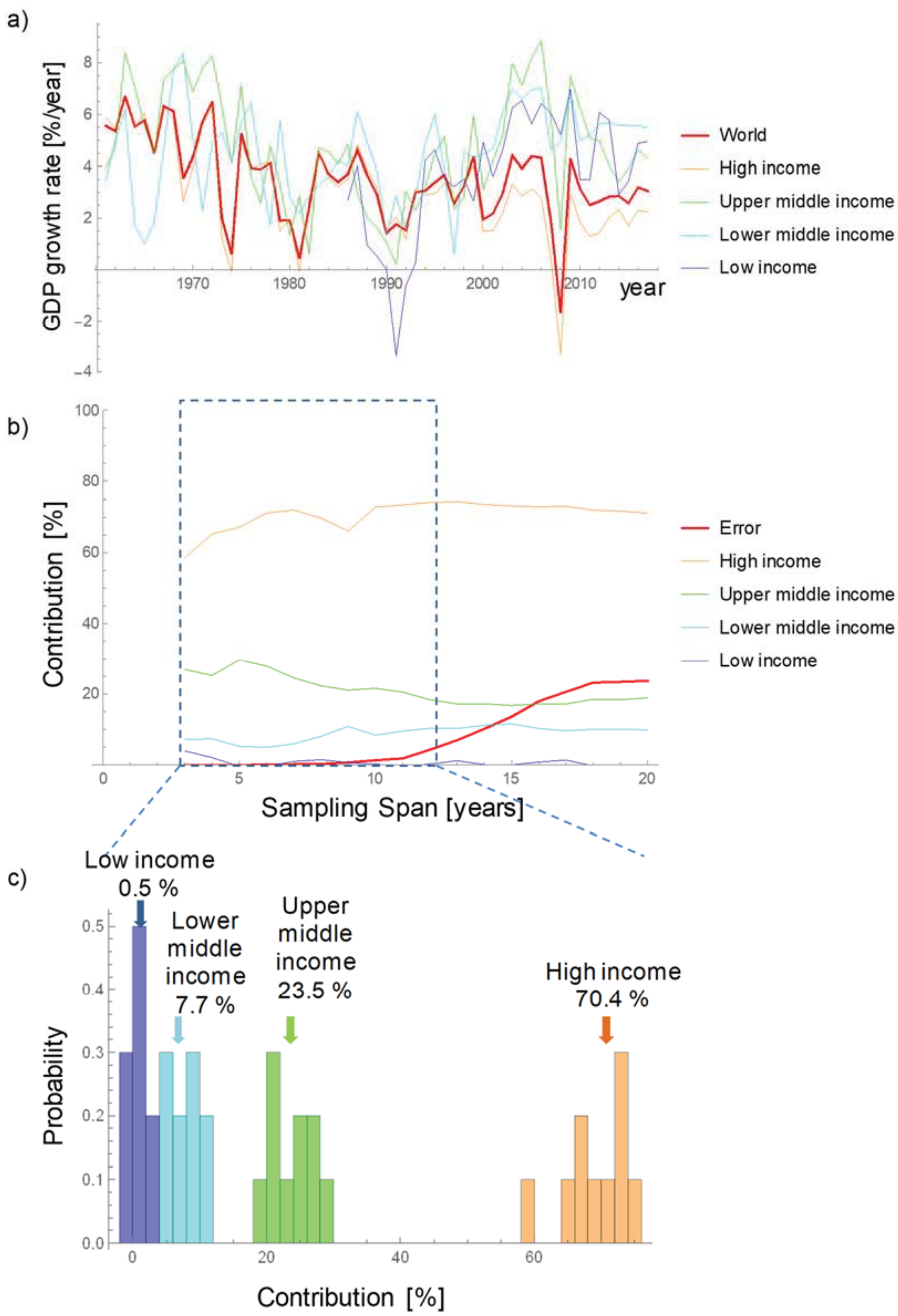


Figure 24. GDP growth rate of income groups and their contributions to the world growth rate.

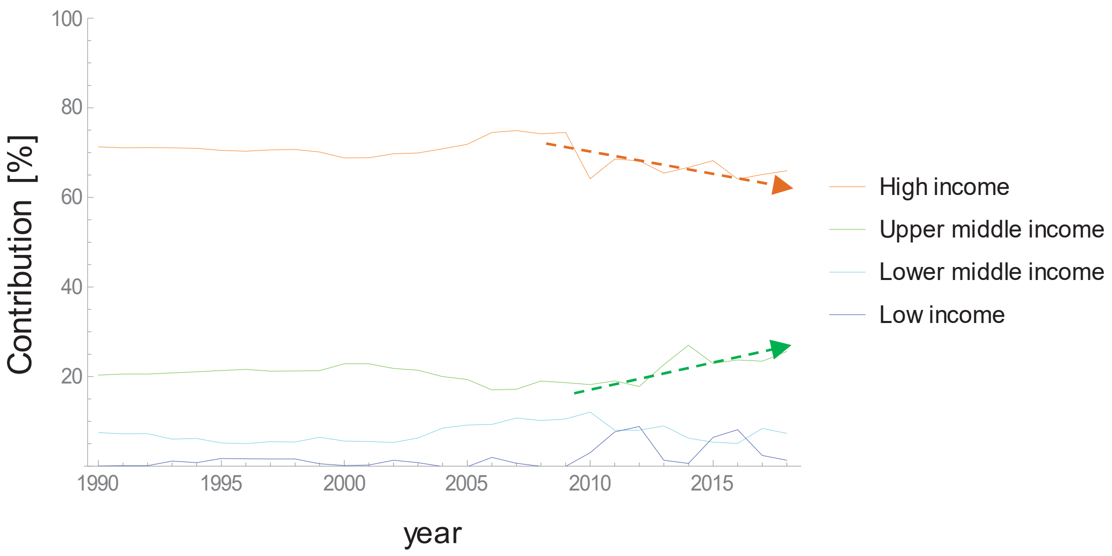


Figure 25. Contribution changes of income groups to world growth rate over years.

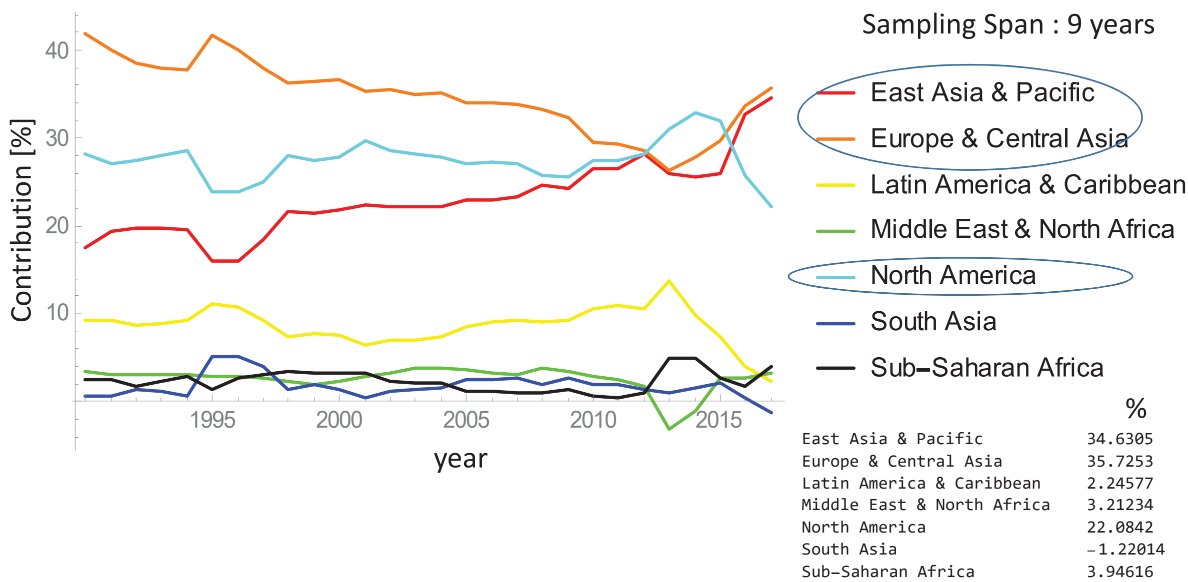


Figure 26. Contribution changes of regions to world growth rate over years.

4.1.2. Growth rate correlations

Correlations between GDP growth rate of individual countries and the world economy growth rate are checked, and the highly correlated results are shown in Figure 27. In the correlation analysis, sampling span is from 2008 to 2017, and the correlation function used in this correlation test is Pearson's correlation coefficient as

$$corr(x,y) \equiv \frac{cov(x,y)}{\sigma_f \sigma_g} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2}\sqrt{\sum_{i=1}^{n}(y_i - \bar{y})^2}} \quad (4.1)$$

where x, y, cov and σ are respectively world growth rate, GDP growth rate of a country, covariance and standard deviation. The correlation coefficient trends of the countries plotted in Figure 28, and the correlation rank does not correspond with GDP ranks of the countries. Rather, small countries seem to be easily influenced by world growth rate, and big countries such as USA seem to be influencing on the world growth rate. This result can be also explained by indirect correlations discussed in Reference 65 and 66. For the next step, pairwise correlations among the countries ranked in top 100 correlations to the world growth rate are checked as shown in Figure 29. The network of the pairwise correlation checks is visualized as shown in Figure 30. The network is formed by the spring-electrical-embedding method minimizing the complexity of the network with the calculations of repulsive force powers. [67] The groups are divided by modularity maximization method of the community theory. [68] This network shows the synchronization and influence of GDP growth rate among countries. They are grouped into 4 groups. The red group in Figure 30 has mostly Eastern European countries, the green group and cyan group have mostly Western European and East Asian countries, and Southern European or West Asia countries belong to the purple group. Particularly, the green group and cyan group represent the core countries of the world economy.

This grouping reveals the economy blocks. The United States, Belgium, Uzbekistan and Bulgaria seem to be the synchronization representatives of their groups.

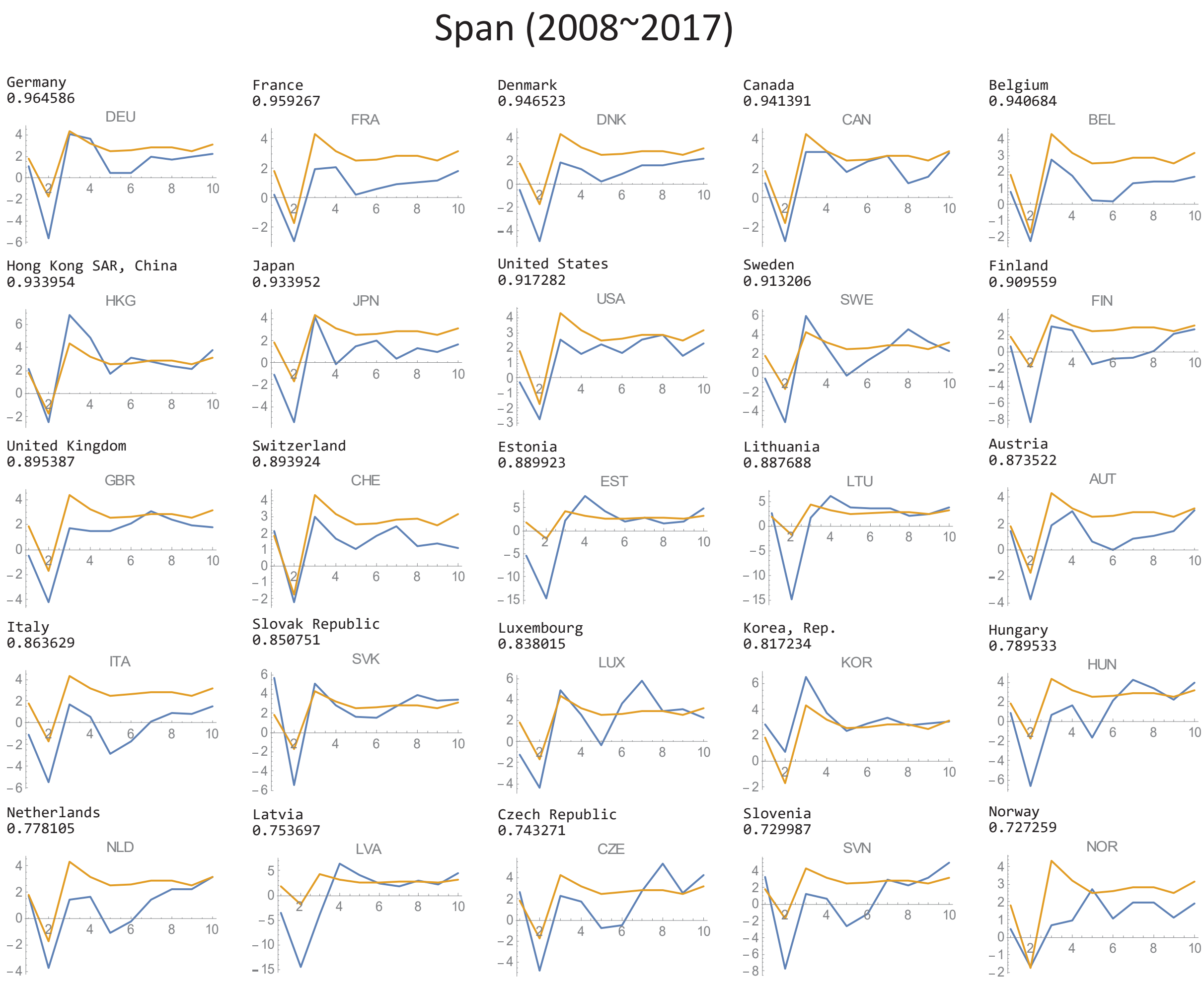


Figure 27. Correlation between world growth rate and that of a country

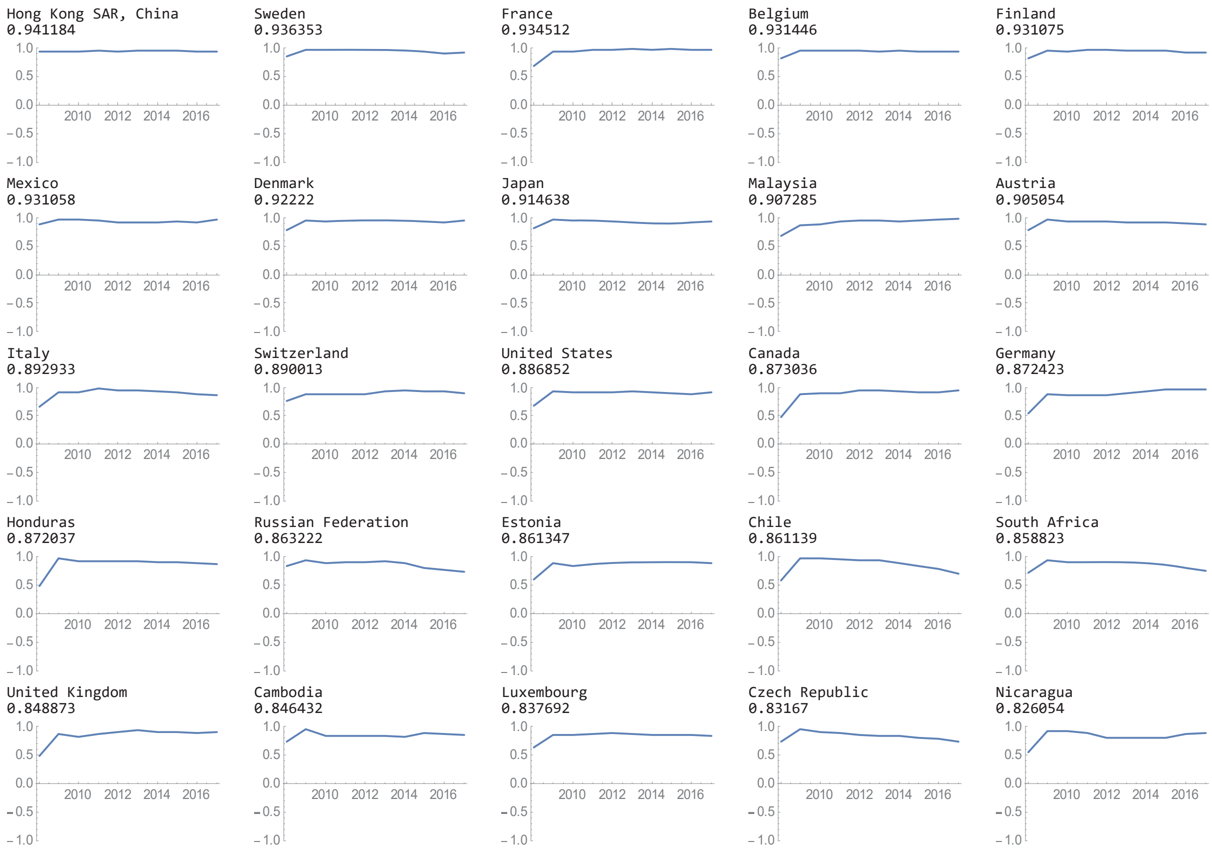


Figure 28. Correlation trends of countries to the world growth rate.

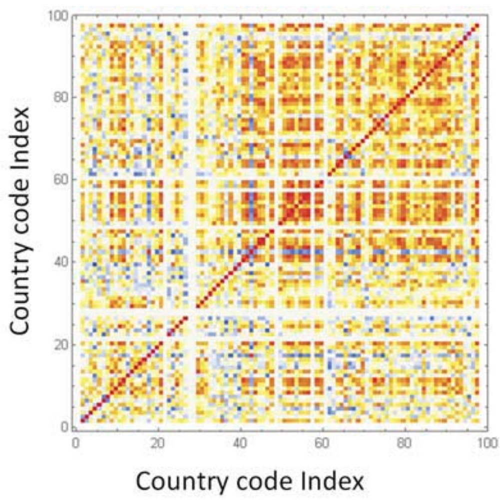


Figure 29. Pairwise correlation checks.

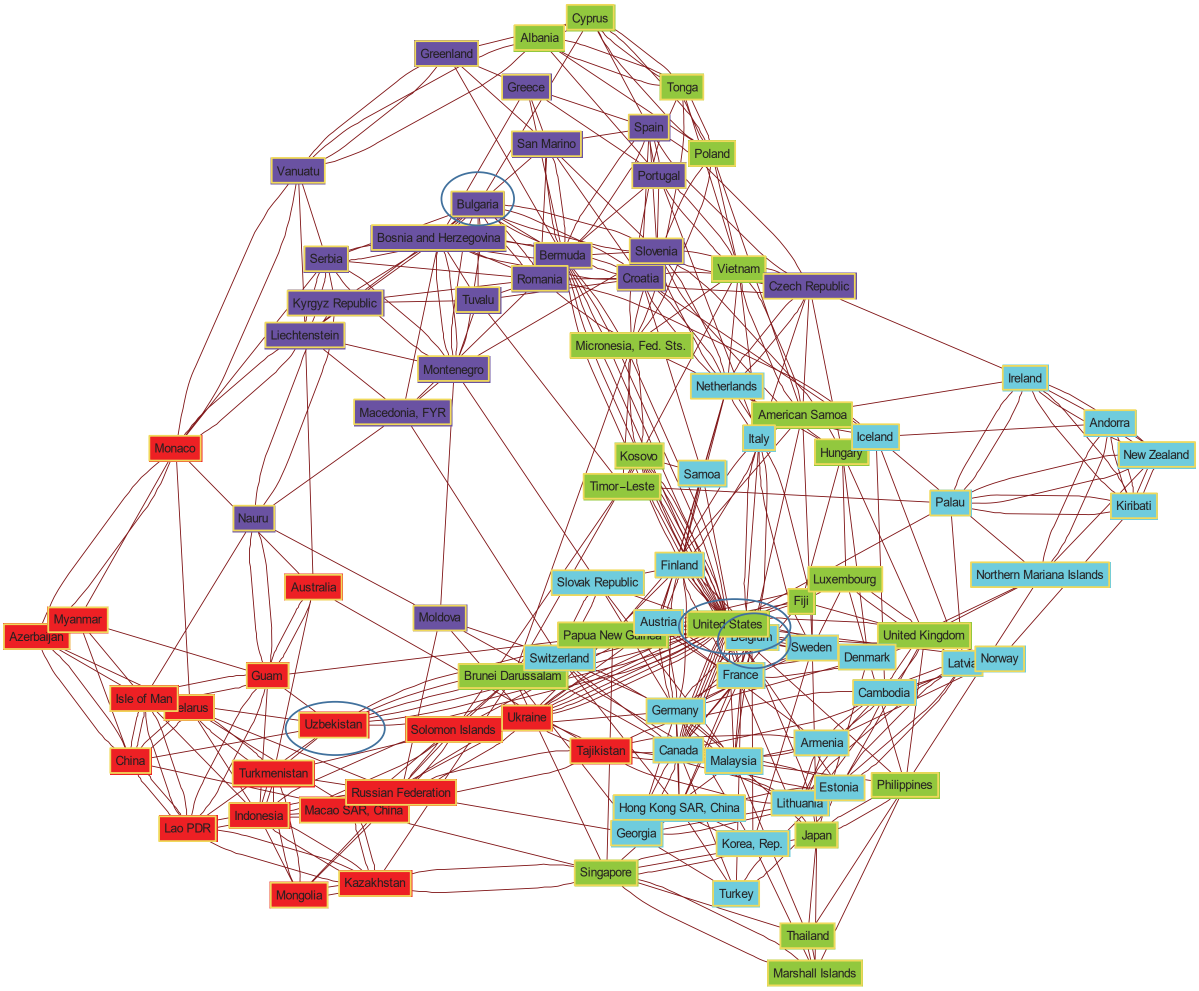


Figure 30. Correlation network

4.2 Commodity trade pattern

As discussed in Section 3.1, trade is one of the engines in GDP growth, which is supported by many evidences as shown in Figure 31. Then, we can have the questions: "Who are the players in global trade?" and "How do they play?". In a word, trade ecology will be investigated in this section. For the analysis, world trade data of World Bank was used.

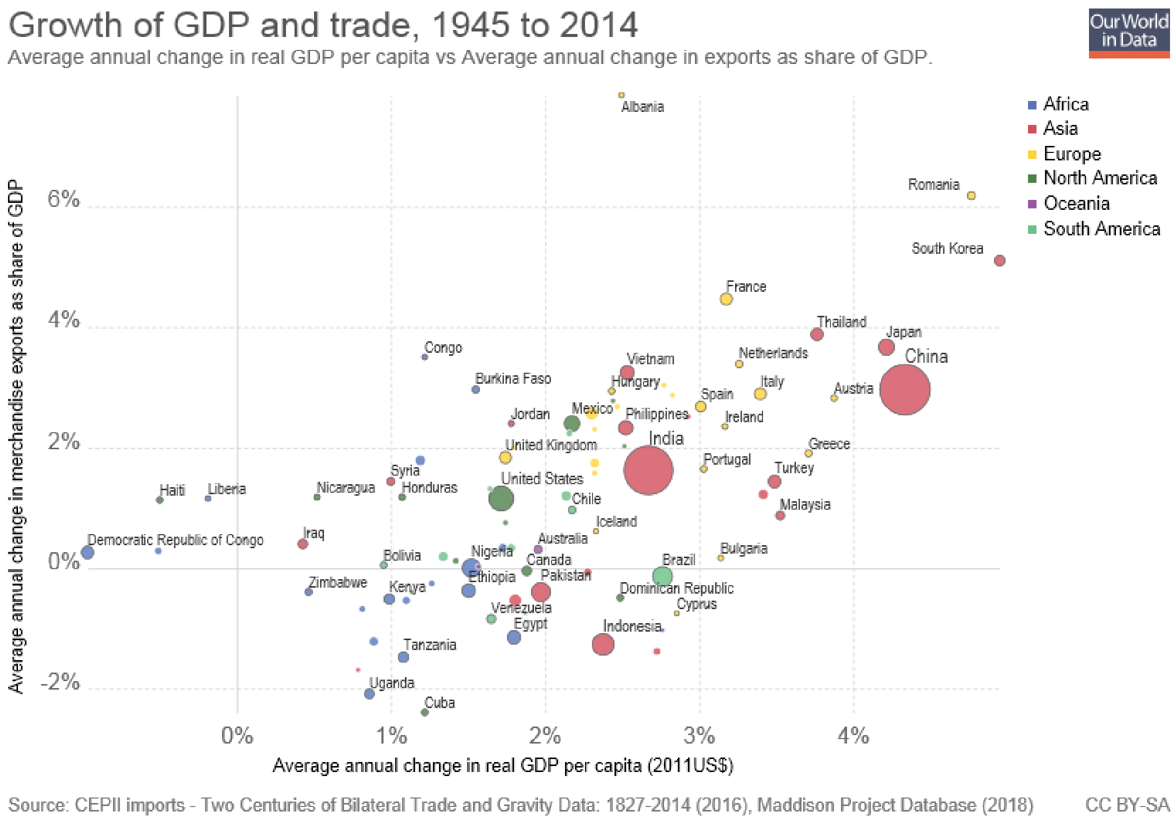


Figure 31. GDP change rate and exports change.

World Bank provides various classification of the trade data. First, the trade volumes by the regional classification are analyzed as shown in Figure 32. Overall trends of exports and imports gets increasing though there are two deeps induced by the US financial crisis and Brexit

& China's market crash as shown in Figure 32 A-1) and A-2). Europe & Central Asia, East Asia & Pacific and North America keep the top 3 ranks in export shares and import shares as shown in Figure 32 B-1) and B-2). East Asia & Pacific shows slightly increasing trend in its export and imports shares over years while that of North America is continuously decreasing as shown in Figure 32 B-1) and B-2). So, these three regions would be important ones in world economy.

For the next, export and import share trends of the countries which have ever registered in the top 10 shares are depicted in Figure 33. In the export shares, China shows the most significant changes. China keeps the first top position after in 2004, and the gap from the second position gets wider. In the import shares, the first top position is United States, and the position has never been replaced. But the first top position seems to be pursued by China steadily. Figure 34 shows the global export and import shares of the top share countries. Top 3 countries among 217 countries cover 30.5 % of the world exports and 28.0 % of the whole global imports. More than 50 % of the whole exports and almost 50 % of the whole imports are traded by only top 10 countries of 217 countries occupy. So, the top 12 countries (the United States, China, Germany, Japan, South Korea, France, Italy, Netherlands, Mexico, Canada and United Kingdom, Belgium) can be regarded as the main players of the world economy.

Now, let us move on to their ways of trading. Top 10 trade countries of the 12 countries are retrieved from World Bank, and they are visualized as shown in Figure 35. The networks are formed by the spring-electrical-embedding method minimizing the complexity of the network with the calculations of repulsive force powers. The node size represents the relative trade volume of a country, and the arrows of the lines represent the directions of the commodity flows: imports and exports. As shown in Figure 35, several countries (Japan, Germany, Hong

Kong, France, United Kingdom and Canada) are gather around the big United States in the 1993 trade network. In the 2003 trade network, China appears in the core country zone, and tight networks of European countries are shown at the left side. The networks might be caused by monetary union of Eurozone. The trade volume of Germany gets bigger than that in 1993 while that of Japan is decreasing. However, the world economy still runs by 6 countries (United States, Germany, China, Japan, France, United Kingdom), and United States keeps the center of the trade network with the first top trade volumes. In the 2018 trade networks, significantly different network figure is depicted as shown in Figure 35 c). Three core countries (China, United States and Germany) occupy the center of the network, and European networks are tightly connected. The first top of the trade volume is replaced by China, and the locations of Japan and South Korea are exchanged respect to the center of the network core zone. So, from the 2018 trade networks, we can find that the recent world economy is managed by China, United States and Germany, and that Germany is the representative of the tight European trade networks.

For the next step to figure out their trade ways, the networks of trade sectors and the countries are illustrated in Figure 36. The sectors represent the trade markets, and Machinery & Electronics is the biggest market as shown in Figure 36. Fuels, Chemicals and Transportation are shown as the core markets besides Machinery & Electronics. The core countries all can be regarded as pro-sumer since they participate in the core markets bi-directionally. This feature can be explained by free market tendency of GVC [19], and the sector classification is too rough to figure out the detail trade flows.

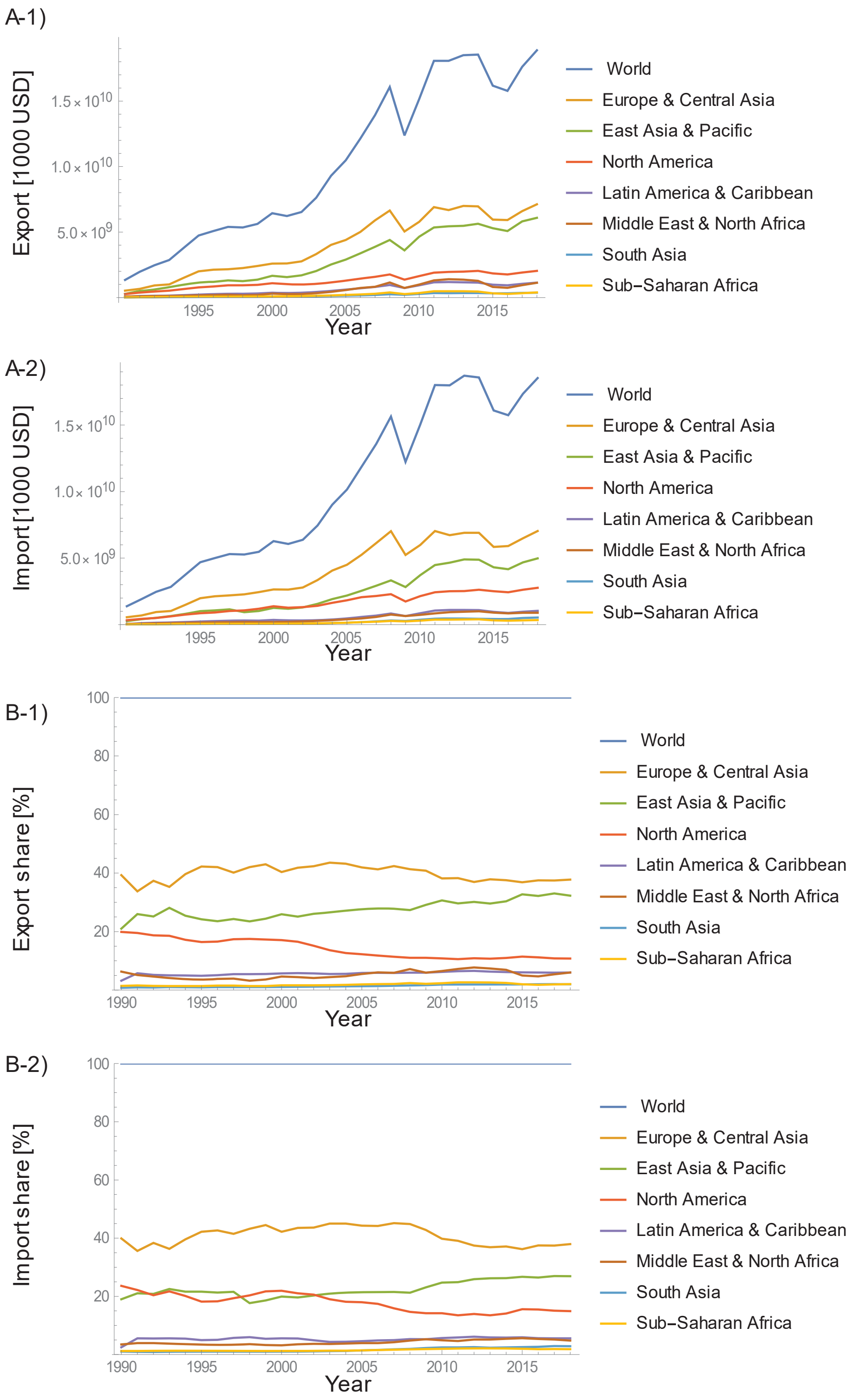


Figure 32. Exports and imports of regions in thousand USD (A-1, A-2) and in percent (B-1, B-2).

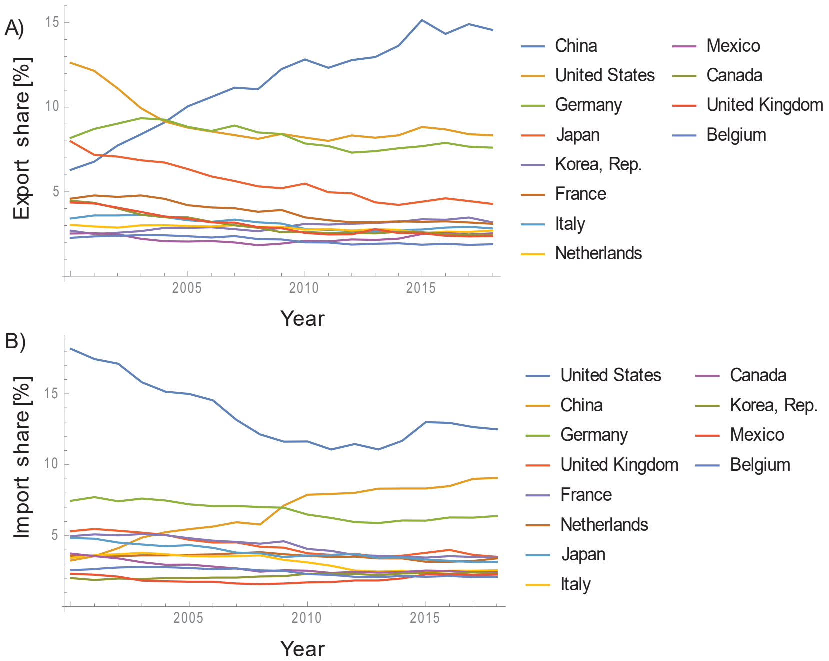


Figure 33. Global export and import shares of the countries registered in top 10 ranks.

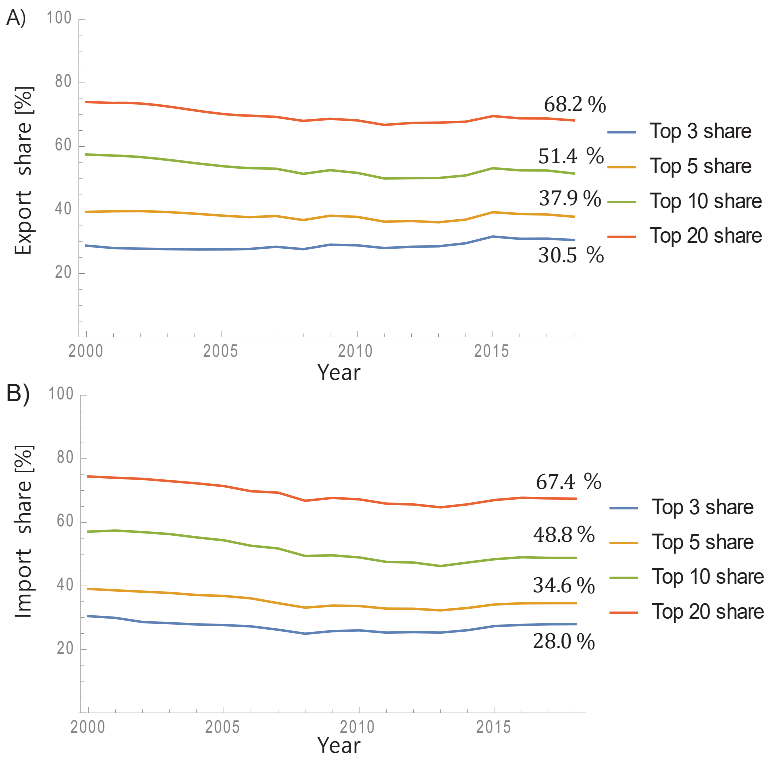


Figure 34. Global export and import shares of the top share countries

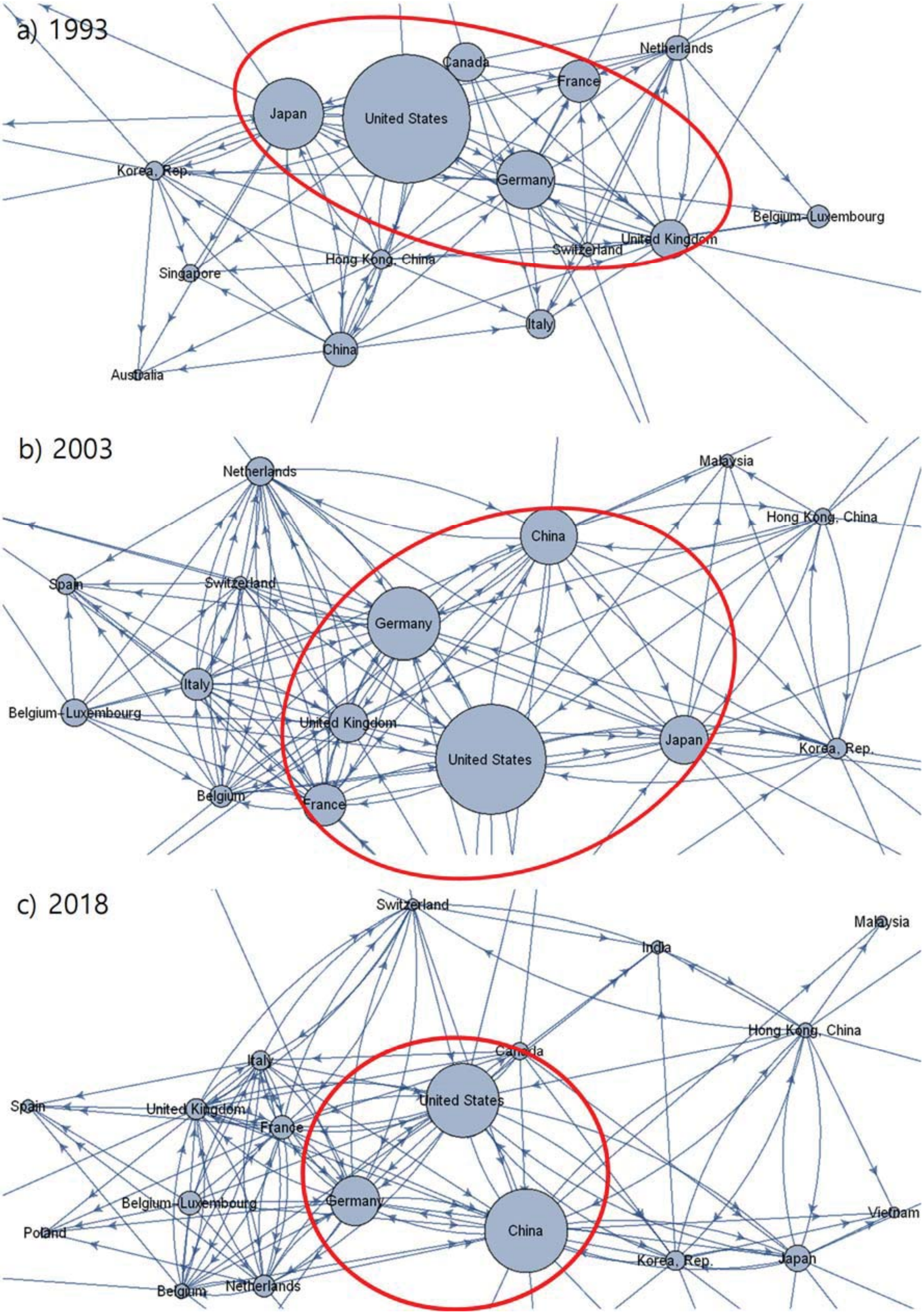


Figure 35. Trade networks in a) 1993, b) 2003 and c) 2018.

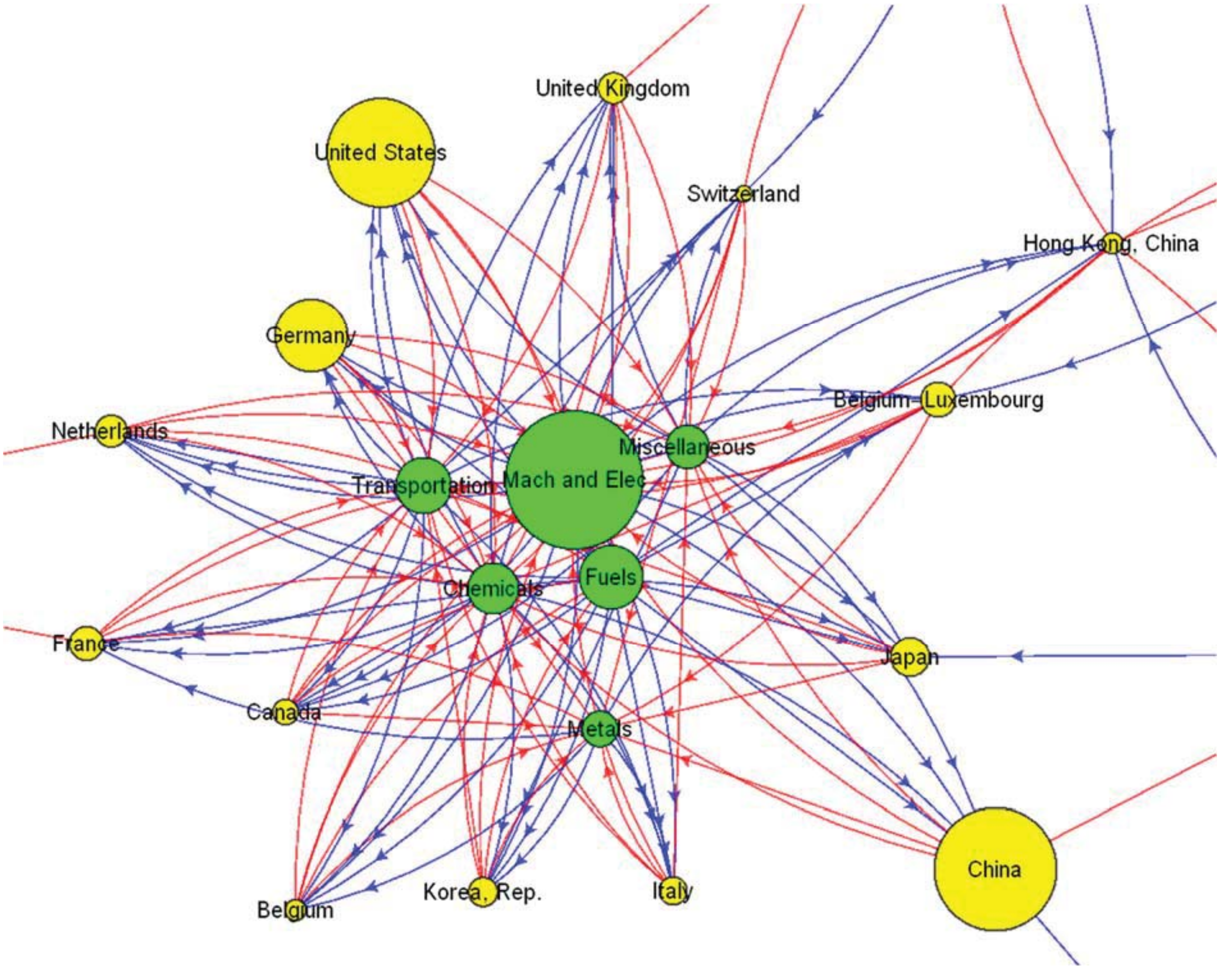


Figure 36. Networks between trade sectors and the 12 countries in 2018.

4.3 Capital Flow Pattern

As discussed in Section 3.1, a financial market is the other channel for international connections. International capital inflow supports long-term growth however it accompanies short-term risks since large-scale capital inflows can overheat the host country's economy. And the bubbles from the over-heating can end with its busting and economic collapse. [73] The capital flow in a financial market can be classified into debts and foreign direct investments (FDIs). In this chapter, national debts and FDIs will be examined to show patterns of the capital flows. Figure 37 shows debt-to-GDP ratios over the world, and Figure 38 shows debts, GDPs and debt-to-GDP ratios of G20 members. The data was retrieved from countryeconomy.com and World Bank. Japan has the highest debt ratio at 238 %, which is almost twice of the second place, Italy. In debt size, the United States ranks first and second ranks Japan. Euro Area followed Japan. China has relatively small debts compared with its GDP. Most advanced economies have around 70 % ~ 100 % of debt ratios. As for the examination of annual net lending ratio to GDP, advanced economies, middle-income economies and low-income economies have borrowed over the five-year period from 2014 to 2018. The average borrowing ratios to the GDPs are respectively 2.65 %, 4.39% and 3.89 % as shown in Figure 39. Over the past five years since 2018, most countries have run a deficit.

Most important thing of the capital flow pattern may be a capital flow network. However, unfortunately, open network data of national debts could be found. Instead, FDI network data [74, 75] was open in OECD database though the FDI data does not cover all countries. Capital flow can be inferred from the FDI networks since debts flow has similar characteristics to FDI flow [73]. The net FDI networks of the OECD members and some others (China and Mexico) are shown in Figure 39. The networks are built with net FDI flows over 1 trillion USD from

2014 to 2018, and the line width presents its relative FDI volume, and the arrows present the directions of FDI flows. Circles indicate countries, and the circle sizes mean the country's total net FDI volume. Red circles and blue circles refer respectively positive net FDI (investing countries) and negative net FDI (invested countries). The data of the networks is listed on Table tb1 of Appendix. In FDI networks, top 2 players are the United States and Japan. The semi-peripheral countries are the United Kingdom, Germany and Luxembourg. Considering the United States and China are the top 2 players in the commodity trade networks in Figure 35 c), the net FDI networks seem to be quite different from the commodity trade networks. The most distinct thing is that Japan has a very tight link with the United States, and the net FDI volume of Japan is the biggest in the networks. Although China has an investing position of the net FDI networks, it is positioned in the peripheral countries. South Korea seems to be placed at the border side of the network map, and its linkage is so weak considering that European countries even with small net FDI volumes have multi-connections. Although net FDI networks ought to be used as the capital networks due to the lack of full capital flow data including debt networks, net FDI networks seem to reveal the capital linkages among countries and the positions in the capital networks. This net FDI networks can be used to connect finance markets in world economy simulations.

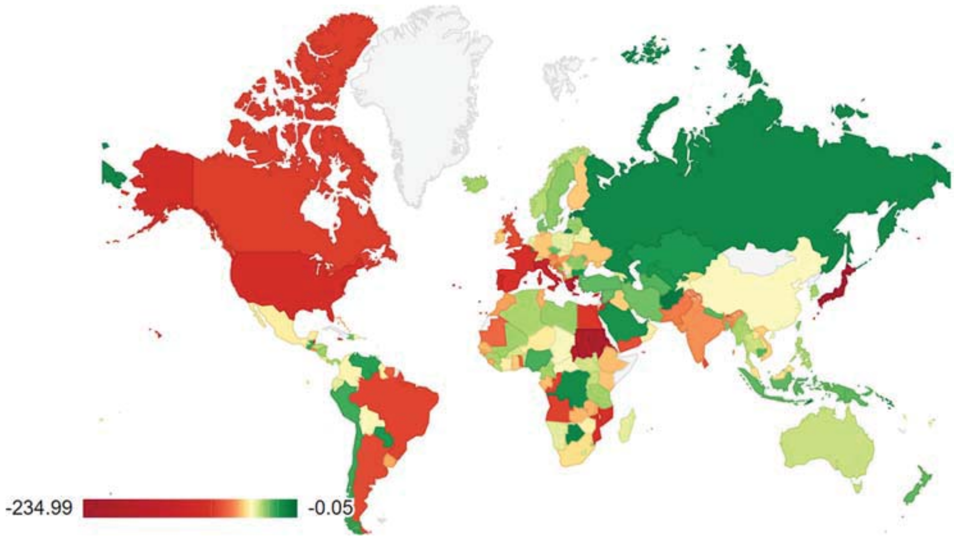


Figure 37. Debt-to-GDP ratio map [%] retrieved from "countryeconomy.com".

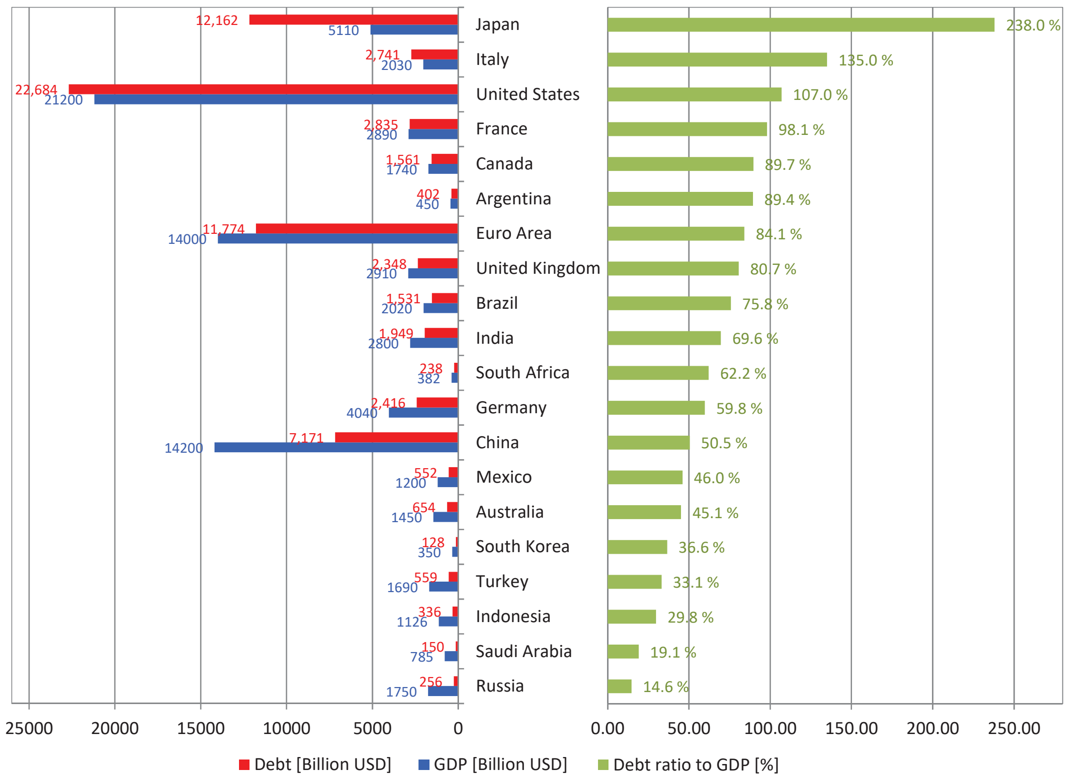


Figure 38. Debt, GDP and debt ratio of G20 members.

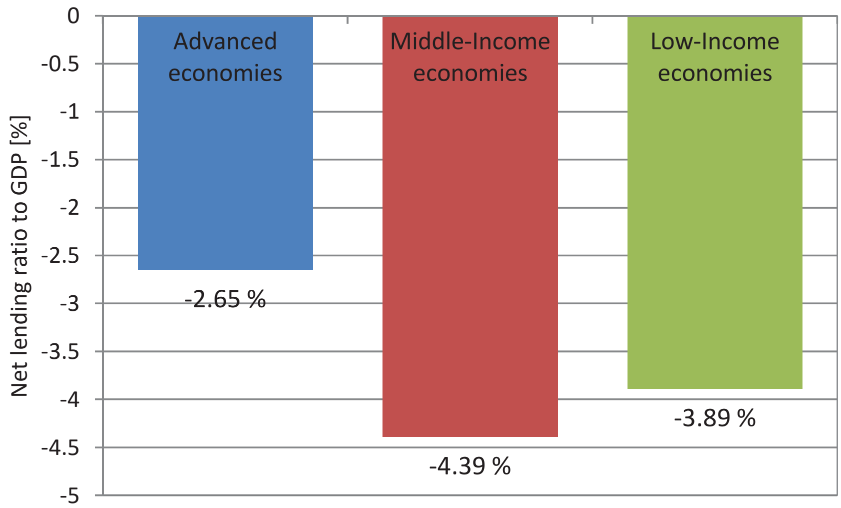


Figure 39. Average net lending ratio to GDP from 2014 to 2018.

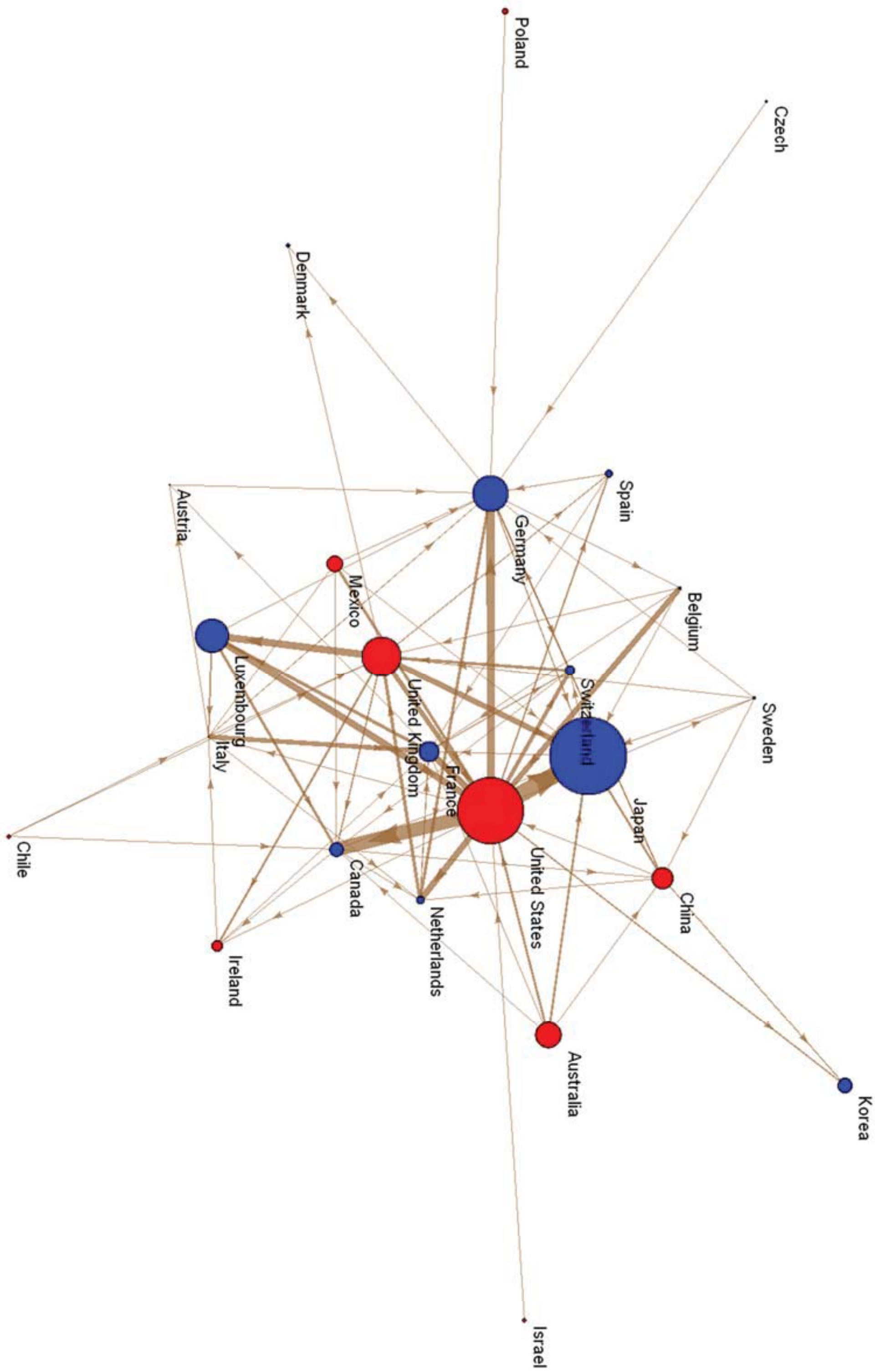


Figure 40. Average net FDI flow from 2014 to 2018.

5. Economy Crises

In the previous chapter, economic networks were examined with real data of goods and finance trades. The networks will be applied to pile the models of Chapter 3 for the global economic networking. The models seem to be suitable to apply the linkages in goods markets and finance markets. In this chapter, we will examine if the mathematical models of the Chapter 3 can depict economic crises. Checking the applicability to the crisis will show the limitation or weak points of the models. So, the features of economic crises will be examined through reviewing 3 big crises (Great depression, 1997 Asian financial crisis and Subprime mortgage crisis). The discussion about the model compatibility for the crises will be followed.

5.1 Great depression

The Great depression indicates the global economic depressions starting from the United States during 1930s. It lasted about 10 years, and it was the longest and deepest depression over the world in the 20th century. [69] It started with an abrupt big fall in stock markets on October 29, 1929, and the big fall is the result of a panic sell-off induced by a deflation in asset and commodity prices, big drops in demand and credit, and trade disruptions. Huge unemployment and impoverishment followed the panic behavior. The worldwide GDP between 1929 and 1932 was estimated to have decreased by 15 %. Considering that worldwide GDP fall during the Great recession in 2009 is less than 1%, the 15 % fall of worldwide GDP is anomalous. During the period, the international trades were reduced to be half, the unemployment in the United States rose up to 23 %. The depression was ended by World War II. [69]

As for the causes, insufficient demands and reductions of money supply argued respectively by Keynesians and monetarists are currently accepted by modern economists. [70] A stable economic growth is referred to be achieved by the macroeconomic aggregations of money supply and aggregate demands, and the roles of the government and the central bank in the markets are now considered to be significant. [71] Although there is no consensus among economists for the escaping force from the Great depression, Roosevelt's New Deal policies are regarded to be one of the recovery forces. [72] The most common view for the recovery forces is that the World War II ended the Great Depression. The government's expenditure on the war is regarded to be the causes of the recovery, and the mobilization of man power for the war ended unemployment. Massive expenditure for the war doubled economic growth rates in the United States.

5.2. 1997 Asian financial crisis

The Asian financial crisis is referred to a financial crisis in East Asia and Southeast Asia in July 1997. It was started in Thailand after baht floating by Thai government due to lack of foreign currency. This induced immediate capital flight, and an international chain reaction followed. Moreover, foreign debt of Thailand reached almost bankrupt level. [77] As the crisis spread, most of Southeast Asia experienced their currency depreciations, the devaluations in their stocks and assets and steep rises in their private debt. [78] This shock had the most serious impact on Indonesia, South Korea, and Thailand. Hong Kong, Malaysia and Philippines were affected while mainland China, Singapore, Taiwan, Vietnam and Japan were less affected. Nevertheless, demand and confidence losses were inevitable throughout the Asia region. [78]

The causes are referred to be credit bubbles and fixed currency exchange rates. The economies of Thailand, Malaysia and Indonesia bred economic bubbles with their hot money from crony capitalism, and more and more money were required to maintain the bubbles. [79] Finally the development money went uncontrollable due to the rapid interest rise of the supplied money. In the mid-1990s, fixed exchange rates of the countries accelerated external borrowing and expose foreign exchange risk in both the financial and corporate sectors. So, the distorted incentives between lenders and borrowers resulted in large quantities of credit and in highly leveraged economic climate. The panic behaviors of the lenders led to refuse to roll over the short-term loans, and those induced the credit withdrawals. To prevent capital flight, high domestic interest rates and comparative fixed exchange rates were kept by governments, but those could not be sustained long. The high domestic interest rates are very harmful to the economies, and those polices could not stop capital flight. So, the fixed exchange rates changed into float rates, and the change induced the depreciation of the currencies. Finally, bankruptcies and financial crises were followed. [78] This havoc ended after intervention of the International Monetary Fund (IMF). A series of bailouts from IMF were supplied to rescue the countries in troubles since the crises could contage the world economies. [80] In a short word, this Asian financial crisis is a national level bank-run after credit bubbling.

5.3 Subprime mortgage crisis

The subprime mortgage crisis beginning from the United State is a worldwide financial crisis between 2007 and 2010. [81] The collapse of a housing bubble induced a significant decline of house prices, and the decline blocked refinancing in the mortgage. It leaded to mortgage delinquencies. This caused the devaluation of housing–related securities. Householders had to reduce their spending. The devaluation was transmitted to the financial market through the losses of the banks and the overall industry due to capital depletion in the banks. Its detail domino propagation from the collapse of housing bubble to the overall industries is shown in Figure 41. The crisis affected the worldwide financial markets due to the security nature of the globally integrated financial markets. The housing bubble was based on excessive financing mortgage-backed securities (MBSes) and collateralized debt obligations (CDOs). The excessive financing was possible under the contributions of financial institutions, regulators, credit agencies, government housing policies, and consumers. The rises of subprime lending and housing speculation were referred to the main negative origins of the excessive financing. [82] This crisis spread out to the developed economies, particularly in North America, South America and Europe, and the great recession followed it. It is referred to be the most severe economic and financial meltdown since the Great Depression by IMF. It lasted over 19 months, and it ended officially on June 2009. The Great Recession seem to be similar to the Great Depression, but the period is comparatively short due to active fights of the previous chairman of the Federal Reserve, Ben Bernanke. He implemented polices to pour the huge amount of money supply and to lower the interest rates as low as possible, which had not been taken in the Great Depression. [84, 85]

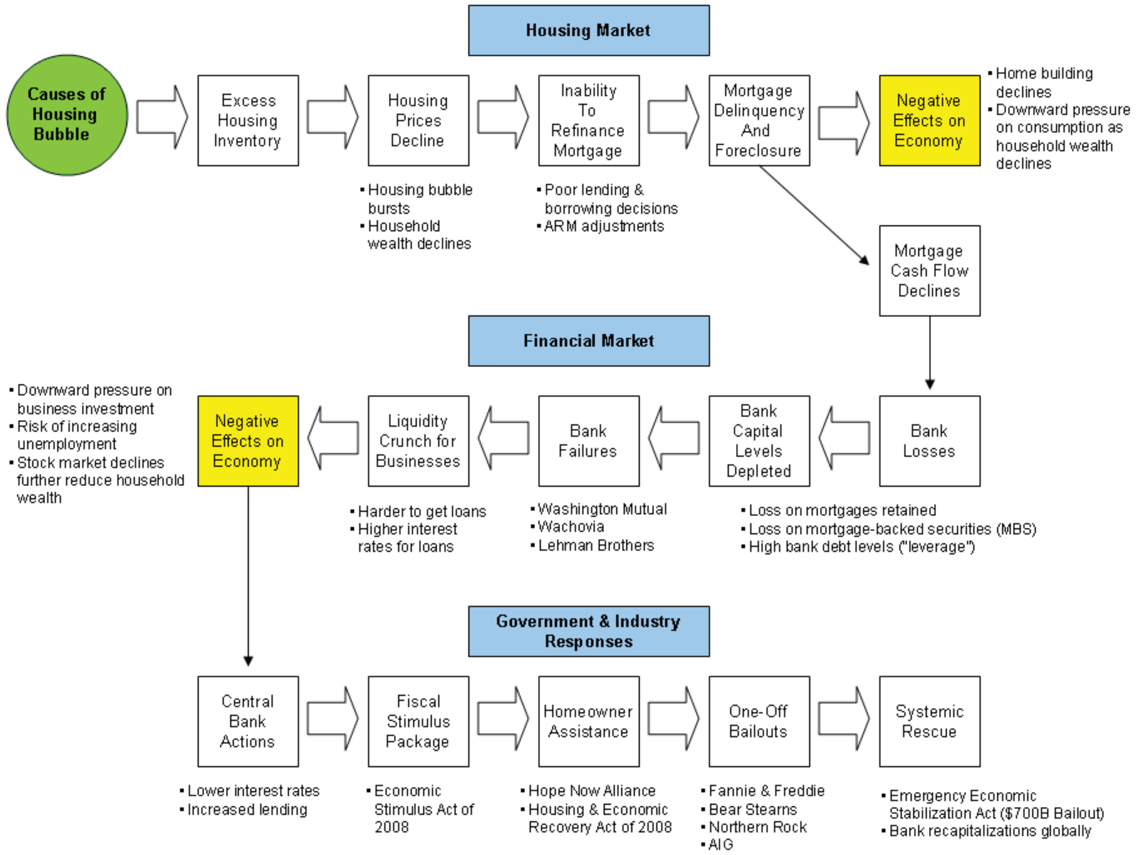


Figure 41. The propagation of the housing bubble collapse [83]

5.4 Origin of economic crises and model compatibility

As we reviewed the cases of the economic crises, most crises have been made in finance markets. It is originated to the nature of financial markets since financial markets can borrow values from the future by credit. The securities financed with future values can fertilize the economic activities, but the credit failures will meltdown the values in the current markets. The credit failure occurs when lenders refuse to extend loan maturity or block new loans due to their fear or the unprofitability truth. The excessive creation of credit fundamentally comes from information inequality. Due to the structural linkage of financial goods, the losses of the credit failure will spread into the related securities or its whole financial market. The fall of capitals in financial markets will despoil the vitality of the whole economies. The economies go into the deflation process.

Another scenario starting from goods markets is possible. Devaluation of goods due to sudden changes in value chains or abrupt crash of consumptions will lower the business productivity of the goods, and that will lose its credit of the related securities. The related securities will be liquidated, and the overall credits in the financial markets will be reduced, and the interest rate will rise sharply. These financial environments will be harmful to business in goods markets, and finally the economies will take deflation processes.

So, abrupt falls in one of goods, financial and labor markets will induce the credit failure. The fear of the market participants will accelerate and amplify the amount of the credit failure. When the amount of the credit failure exceeds beyond a certain sustainable level of the economies, credit withdrawal will happen in chain reactions, and its result is the big fall of the economies and deflations. To present the crisis, credit creation/liquidation, panic behavior and devaluation should be included in the model since those three factors are the key components of the crisis. The models based on the economic textbooks have just price adjustments. So, devaluation can be presented by the classic model. The player's prediction added on the model in the Chapter 3 can include the panic behavior. However, the credit creation and liquidation are not reflected on the model though the monetary policies of the government can be presented on the model. The model needs to include the financial algorithms for security creation and liquidation on the financial market modeling, and the modification of the financial market can complete the economic modeling. Due to the complexity of the financial goods, the algorithm about the credit creation was not tried to be developed in this thesis, but the liquidation or credit creation can be presented in the model by applying depreciation or appreciation factor to savings, respectively.

6. Summary

Economies need to be treated in engineering area since the economies get more complicated and faster. This thesis aims to provide the mathematical models of the world economy, and this work reveals the mechanism of the world economy. From the macroscopic view, the world system can be segmented into 3 groups (core countries, semi-peripheral countries and peripheral countries). The core countries occupy a dominant position with higher productivity than the periphery countries. Their roles are reflected on the trade patterns, the trade patterns are based on global value chains (GVC). The GVC is cross-geographical or cross-national fragmented production process. The economies are known to have their own business cycle, and the causes of the business cycle do not seem to converge to a single explanation. Under open economy with the trade and capital flow, economy of a country can be affected by the external economy. Due to globalization or global value chains, the impact from the external economy gets severe, and that has been shown as economy synchronizations. The pathology of the economy synchronizations has been studied as financial contagions. Then, we are facing the question; “How does the world economy work?”. The exploration has begun with the theories of the economic textbooks in Chapter 3. The theories are integrated mathematically in a single tone, while they are fragmented into micro-economics, macro-economics, and international trade. Adding players’ prediction behavior on the previous theories, the theories are able to present transient reactions of the markets. Combining newly derived market equations, international trade patterns could be precisely predicted. So, the adjustment of trade patterns can be calculated when price level of a country changes. This theory integration of the economic textbooks can also provide the intrinsic reasons of the economic phenomena. So, the trade model of this thesis reveals the reason of Gravity theory of the international trade as the maximizing efficiency of the economies.

To simulate the real world economies, the derived market models of a country should mirror the real markets of the country including the real international networks. So, Chapter 4 shows the real network patterns of the world economy. In the examination of GDP growth rates, high income economies determine 70.4 % of the world growth rate. The contribution from upper middle income economies to the world growth rate gets slowly higher while that from high income economies gets slightly lower after 2010. In the correlation test of the GDP growth rates, the countries are classified into 4 groups, and the United States, Belgium, Uzbekistan and Bulgaria seem to be the synchronization representatives of their groups. In commodity trade analysis, more than 50 % of the whole exports and almost 50 % of the whole imports are traded by only top 10 countries of 217 countries. So, the top 12 countries (the United States, China, Germany, Japan, South Korea, France, Italy, Netherlands, Mexico, Canada and United Kingdom, Belgium) can be regarded as the main players of the world economy. Machinery & Electronics sector is the biggest market of the global trade. The core countries all can be regarded as prosumer since they participate in the core markets bi-directionally. The commodity trade network in 2018 seems to be ruled by 3 countries (the United State, China, and Germany). In the analysis of capital flows, most advanced economies have around 70 % ~ 100 % of debt-to-GDP ratios and Japan has the highest debt ratio at 238 %, which is almost twice of the second place. Due to the lack of debt network data, net FDI networks are used to figure out the capital flow network. The United States and Japan are figured out to be respectively the biggest investing country and invested country in the world. These two countries have almost 3 times bigger capital flows than the United Kingdom at the 3rd biggest capital flows, and they together have a very tight network. In capital flow networks, Germany, the United Kingdom and Luxembourg are placed as the semi-peripheral countries. The net FDI flow of China is not so significant in the net FDI networks while China has the 2^{nd} largest country in the commodity networks.

To examine the limitation of the models, their applicability to economy crises is checked. The model based on those of economic textbooks shows its weak points that the excessive credit creation process is not intrinsically embedded, while the panic behaviors of the players can be reflected. So, the model needs to be complemented by the credit process of the financial markets. This thesis presents the economic modeling of national economy through integration of the conventional economic models and its applicability to the trade pattern analysis with piling the models. For building the simulator of the world economy, GDP patterns of the countries, commodity trade networks and capital flows are examined with the economic data.

Reference

[1] Barr, A. and Feigenbaum, E. A. (Eds.). (2014). *The handbook of artificial intelligence* (Vol. 2). Butterworth-Heinemann.

[2] Sutskever, I. and Hinton, G. E. (2008). Deep, narrow sigmoid belief networks are universal approximators. *Neural computation*, *20*(11), 2629-2636.

[3] Araújo, S. and J. Oliveira Martins (2009), The Great Synchronisation: what do high-frequency data statistics tell us about the trade collapse?. VoxEU.org, 8 July 2009.

[4] Arrow, K. J.; Debreu, G. (1954). The Existence of an Equilibrium for a Competitive Economy. Econometrica. 22 (3): 265–290.

[5] McKenzie, Lionel W. (1959). On the Existence of General Equilibrium for a Competitive Economy. Econometrica. 27 (1): 54–71.

[6] Debreu, G. (1959). *Theory of Value*. New York: Wiley. p. 98.

[7] Immanuel Wallerstein (2004). World-systems Analysis, In World System History, ed. George Modelski, in Encyclopedia of Life Support Systems (EOLSS), Developed under the Auspices of the UNESCO, Eolss Publishers, Oxford, UK.

[8] Immanuel Wallerstein (1989). The Modern World-System III, San Diego: Academic Press.

[9] I. M. Wallerstein and S. R. I. Wallerstein (2004). World-systems analysis: An introduction. Duke University Press.

[10] Immanuel Wallerstein (1974). The Modern World-System I: Capitalist Agriculture and the Origins of the European World-Economy in the Sixteenth Century, New York: Academic Press.

[11] Immanuel Wallerstein (1992). The West, Capitalism, and the Modern World-System, Review 15 (4), 561-619.

[12] Alvin Y. So (1990). Social Change and Development: Modernization, Dependency, and World-Systems Theory, Newbury Park, London and New Delhi: Sage Publications. pp. 169–199.

[13] Abu-Lughod, J. L. (1989). *Before European hegemony: the world system AD 1250-1350*. Oxford University Press, USA.

[14] Frank, A. G., Gills, B., & Gills, B. K. (Eds.). (1996). *The world system: five hundred years or five thousand?*. Psychology Press.

[15] Korotayev, A. (2005). A compact macromodel of world system evolution. *Journal of World-Systems Research*, *11*(1), 79-93.

[16] G. Gereffi (1996). Global commodity chains: new forms of coordination and control among nations and firms in international industries. Competition & Change, 1(4), 427-439.

[17] Dixit, A., & Norman, V. (1980). *Theory of international trade: A dual, general equilibrium approach*. Cambridge University Press.

[18] H. Escaith, and S. Miroudot (2016). Industry-level competitiveness and inefficiency spillovers in global value chains. In *24th International Input-Output Conference, Seoul, Korea*.

[19] G. Gereffi, J. Humphrey and T. Sturgeon (2005). The governance of global value chains. *Review of international political economy*, *12*(1), 78-104.

[20] Frederick, S. (2019). Global value chain mapping. In *Handbook on Global Value Chains*.

Edward Elgar Publishing.

[21] Todeva, E. (2006). *Business networks: strategy and structure*. Routledge.

[22] Shi, Y., & Gregory, M. (1998). International manufacturing networks—to develop global competitive capabilities. *Journal of operations management*, *16*(2-3), 195-214.

[23] Gereffi, G. (1999). International trade and industrial upgrading in the apparel commodity chain. *Journal of international economics*, *48*(1), 37-70.

[24] Todeva, E., & Rakhmatullin, R. (2016). Industry Global Value Chains, Connectivity and Regional Smart Specialisation in Europe. *An Overview of Theoretical Approaches and Mapping Methodologies (No. JRC102801). Joint Research Centre (Seville site).*

[25] Lee, M. W. (1955). Economic Fluctuations, Richard D. *Irwin, Homewood, IL*.

[26] Schumpeter, J. A. (1954). *History of economic analysis*. Psychology Press.

[27] Korotayev, A. V., & Tsirel, S. V. (2010). A spectral analysis of world GDP dynamics: Kondratieff waves, Kuznets swings, Juglar and Kitchin cycles in global economic development, and the 2008–2009 economic crisis. *Structure and Dynamics*, *4*(1).

[28] Say, J. B., & Biddle, C. C. (1851). *A treatise on political economy*. Philadelphia: Grigg & Elliott.

[29] Keynes, J. M. (1936). *The general theory of interest, employment and money*, London: MacMillan.

[30] Drautzburg, T. (2019). Why Are Recessions So Hard to Predict? Random Shocks and Business Cycles. *Economic Insights*, *4*(1), 1-8.

[31] Cutler, T., Hindess, B., Hussain, A., & Hirst, P. Q. (2013). *Marx's capital and capitalism today* (Vol. 52). Routledge.

[32] Kalecki, M. (1943). Political aspects of full employment 1. *The political quarterly*, *14*(4), 322-330.

[33] Estrella, A., & Mishkin, F. S. (1998). Predicting US recessions: Financial variables as leading indicators. *Review of Economics and Statistics*, *80*(1), 45-61.

[34] Business cycle. (n.d.). In Wikipedia. Retrieved May 7, 2020, from https://en.wikipedia.org/wiki/Business_cycle.

[35] Kose, M. A., Prasad, E. S., & Terrones, M. E. (2003). How does globalization affect the synchronization of business cycles?. *American Economic Review*, *93*(2), 57-62.

[36] Canova, F., & Dellas, H. (1993). Trade interdependence and the international business cycle. *Journal of international economics*, *34*(1-2), 23-47.

[37] Dées, S., & Zorell, N. (2012). Business cycle synchronisation: Disentangling trade and financial linkages. *Open Economies Review*, *23*(4), 623-643.

[38] Stock, J. H., & Watson, M. W. (2005). Understanding changes in international business cycle dynamics. *Journal of the European Economic Association*, *3*(5), 968-1006.

[39] De Pace, P. (2013). Currency union, free-trade areas, and business cycle synchronization. *Macroeconomic Dynamics*, *17*(3), 646-680.

[40] Braun, M., & Larrain, B. (2005). Finance and the business cycle: international, inter-industry evidence. *The Journal of Finance*, *60*(3), 1097-1128.

[41] Crowley, P., & Trombley, C. (2015). Macroeconomic Synchronization and Monetary Unions. European Union Studies Association (EUSA)

[42] Xi, N., Muneepeerakul, R., Azaele, S., & Wang, Y. (2014). Maximum entropy model for business cycle synchronization. *Physica A: Statistical Mechanics and its Applications*, *413*, 189-194.

[43] Groth, A., & Ghil, M. (2017). Synchronization of world economic activity. *Chaos: An Interdisciplinary Journal of Nonlinear Science*, *27*(12), 127002.

[44] Monnet, E., & Puy, D. (2016). *Has Globalization Really Increased Business Cycle Synchronization?*. International Monetary Fund.

[45] Oliveira, J., & Araujo, S. (2009). The Great Synchronisation: Tracking the Trade Collapse with High-Frequency Data. *The Great Trade Collapse: Causes, Consequences and Prospects, CEPR, Londres*.

[46] Belke, A., Domnick, C., & Gros, D. (2017). Business cycle synchronization in the EMU: Core vs. periphery. *Open Economies Review*, *28*(5), 863-892.

[47] Peckham, R. (2013). Economies of contagion: financial crisis and pandemic. *Economy and Society*, *42*(2), 226-248.

[48] Schmukler, S. L. (2004). Benefits and risks of financial globalization: challenges for developing countries.

[49] Dornbusch, R., Park, Y. C., & Claessens, S. (2000). Contagion: understanding how it spreads. *The World Bank Research Observer*, *15*(2), 177-197.

[50] Allen, F., & Gale, D. (2000). Financial contagion. *Journal of political economy*, *108*(1), 1-33.

[51] Lagunoff, R., & Schreft, S. L. (2001). A model of financial fragility. *Journal of Economic Theory*, *99*(1-2), 220-264.

[52] King, M. A., & Wadhwani, S. (1990). Transmission of volatility between stock markets. *The Review of Financial Studies*, *3*(1), 5-33.

[53] Forbes, K. J., & Rigobon, R. (2002). No contagion, only interdependence: measuring stock market comovements. *The journal of Finance*, *57*(5), 2223-2261.

[54] Gagniuc, P. A. (2017). *Markov chains: from theory to implementation and experimentation*. John Wiley & Sons.

[55] Besanko, D., & Braeutigam, R. (2020). *Microeconomics*. John Wiley & Sons.

[56] Baumol, W. J., & Blinder, A. S. (2015). *Microeconomics: Principles and policy*. Nelson Education.

[57] Mankiw, N. G. (2003). Macroeconomics., Worth Publishers.

[58] Blanchard, O. J., Fischer, S., & Blancard, O. A. (1989). *Lectures on macroeconomics*. MIT press.

[59] Krugman, P. R., Obstfeld, M., & Melitz, M. J. (2015). International Economics: Theory and Policy.

[60] Salvatore, D. (2016). *International economics*. John Wiley & Sons.

[61] Indifference curve. (n.d.). In Wikipedia. Retrieved June 10, 2020, from https://en.wikipedia.org/wiki/Indifference_curve.

[62] Taylor series. (n.d.). In Wikipedia. Retrieved June 10, 2020, from https://en.wikipedia.org/wiki/Taylor_series.

[63] Comparative advantage. (n.d.). In Wikipedia. Retrieved June 10, 2020, from https://en.wikipedia.org/wiki/Comparative_advantage.

[64] Heckscher–Ohlin model. (n.d.). In Wikipedia. Retrieved June 10, 2020, from https://en.wikipedia.org/wiki/Heckscher%E2%80%93Ohlin_model.

[65] Schneidman, E., Berry, M. J., Segev, R., & Bialek, W. (2006). Weak pairwise correlations imply strongly correlated network states in a neural population. *Nature*, *440*(7087), 1007-1012.

[66] Ueoka, Y., Suzuki, T., Ikeguchi, T., & Horio, Y. (2008). Efficiency of statistical measures to estimate network structure of chaos coupled systems. In *Proceedings of 2008 International Symposium on Nonlinear Theory and its Applications (NOLTA)*.

[67] Hu, Y. (2005). Efficient, high-quality force-directed graph drawing. *Mathematica Journal*, *10*(1), 37-71.

[68] Chen, M., Kuzmin, K., & Szymanski, B. K. (2014). Community detection via maximization of modularity and its variants. *IEEE Transactions on Computational Social Systems*, *1*(1), 46-65.

[69] Duhigg, C. (2008). Depression, you say? Check those safety nets. *New York Times*, *23*.

[70] Whaples, R. (1995). Where is there consensus among American economic historians? The results of a survey on forty propositions. *The Journal of Economic History*, *55*(1), 139-154.

[71] De Long, J. B. (1990). *" Liquidation" Cycles: Old-Fashioned Real Business Cycle Theory and the Great Depression* (No. w3546). National Bureau of Economic Research.

[72] Eggertsson, G. B. (2008). Great Expectations and the End of the Depression. *American Economic Review*, *98*(4), 1476-1516.

[73] Furceri, D., Guichard, S., & Rusticelli, E. (2011). Episodes of large capital inflows and the likelihood of banking and currency crises and sudden stops, *OECD Economics Department Working Papers*.

[74] Comparative advantage. (n.d.). In countryeconomy. Retrieved June 10, 2020, from.

[75] OECD (2020), Outward FDI flows by partner country (indicator). doi: 10.1787/408bbdcc-en (Accessed on 10 June 2020)

[76] OECD (2020), Inward FDI flows by partner country (indicator). doi: 10.1787/04f8069b-en (Accessed on 22 June 2020)

[77] Veigel, K. (1999). *The Asian Financial Crisis: Facts and Explanations*. diplom. de.

[78] Yamazawa, I. (1998). The Asian economic crisis and Japan. *The Developing Economies*, *36*(3), 332-351.

[79] Hughes, H. (1999). Crony capitalism and the East Asian currency and financial'crises'. *Policy: A Journal of Public Policy and Ideas*, *15*(3), 3.

[80] Fischer, S. (1998, March). The IMF and the Asian crisis. In *Forum Funds Lecture at UCLA* (Vol. 20).

[81] Duca, J. V. (2013). Subprime mortgage crisis. *Federal Reserve History*, *23*.

[82] Albanesi, S., De Giorgi, G., & Nosal, J. (2017). *Credit growth and the financial crisis: A new narrative* (No. w23740). National Bureau of Economic Research.

[83] Subprime mortgage crisis. (n.d.). In Wikipedia. Retrieved May 27, 2020, from https://en.wikipedia.org/wiki/Subprime_mortgage_crisis.

[84] Davis, B. (2009). What's a Global Recession?. *The Wall Street Journal*, *22*.

[85] International, F. M. (2009). *World Economic Outlook: April 2009: Crisis and Recovery*. International Monetary Fund.

Appendix

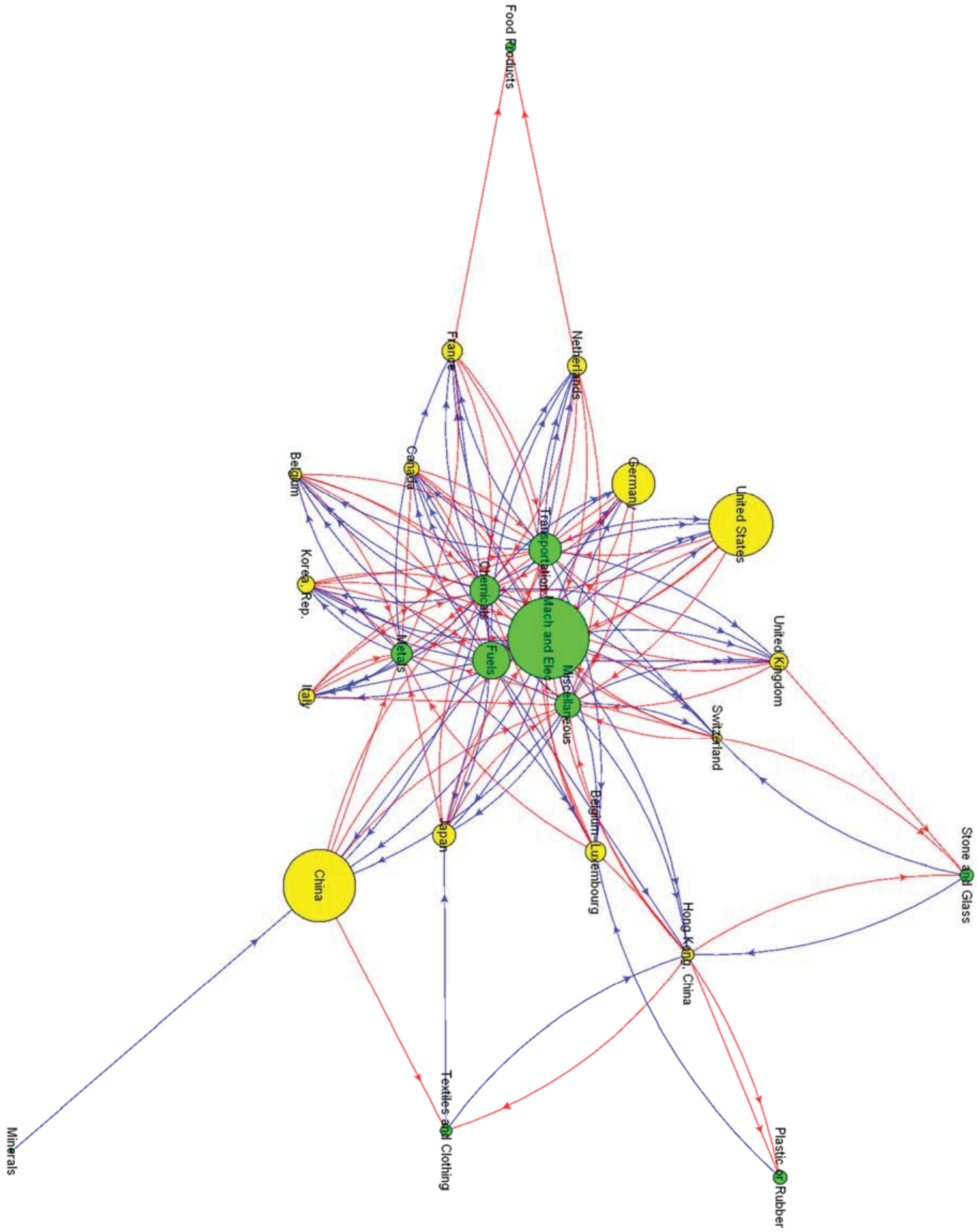


Figure A-1. World trade markets and players in 2018

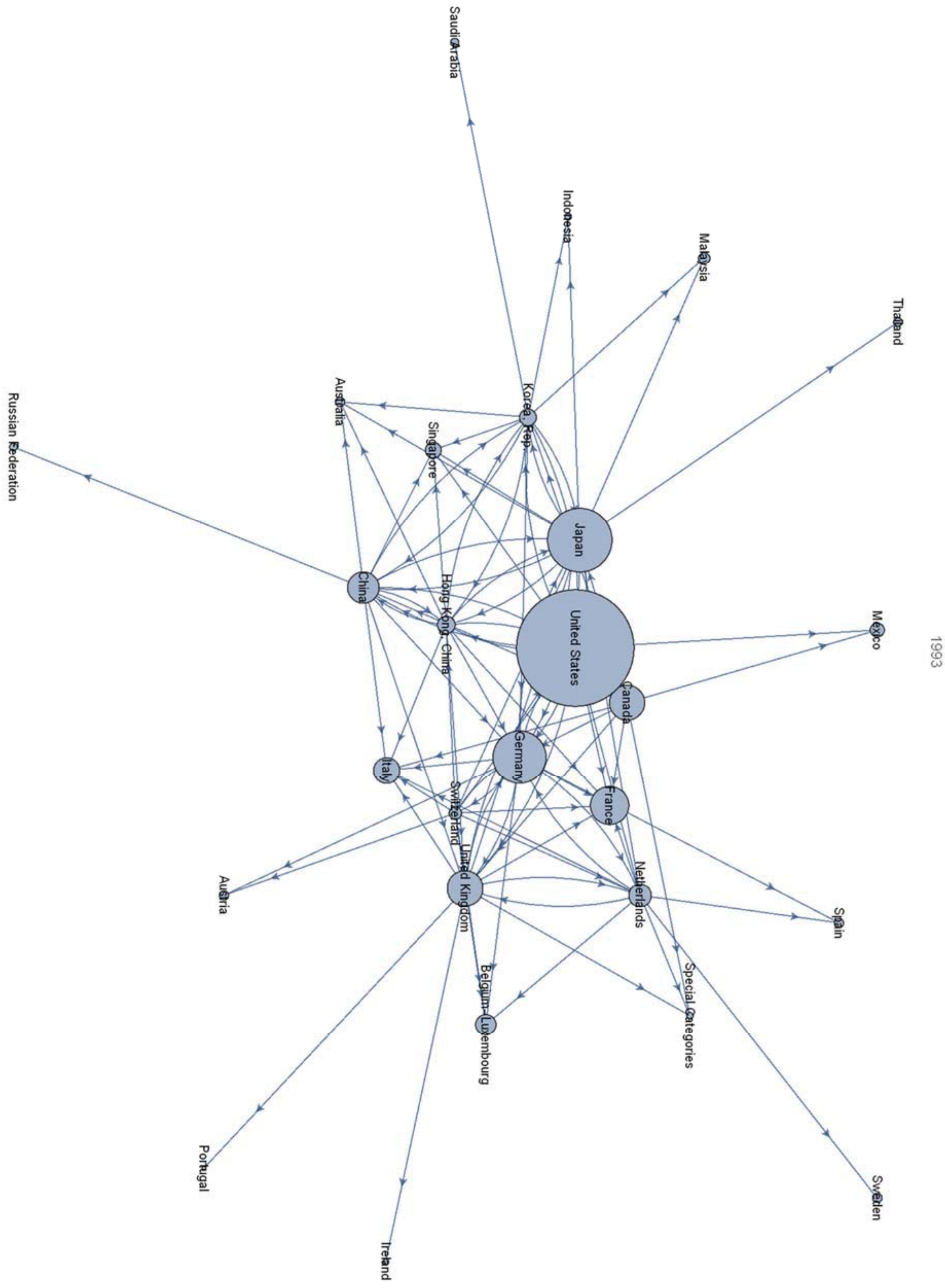


Figure A-2. Trade map in 1993

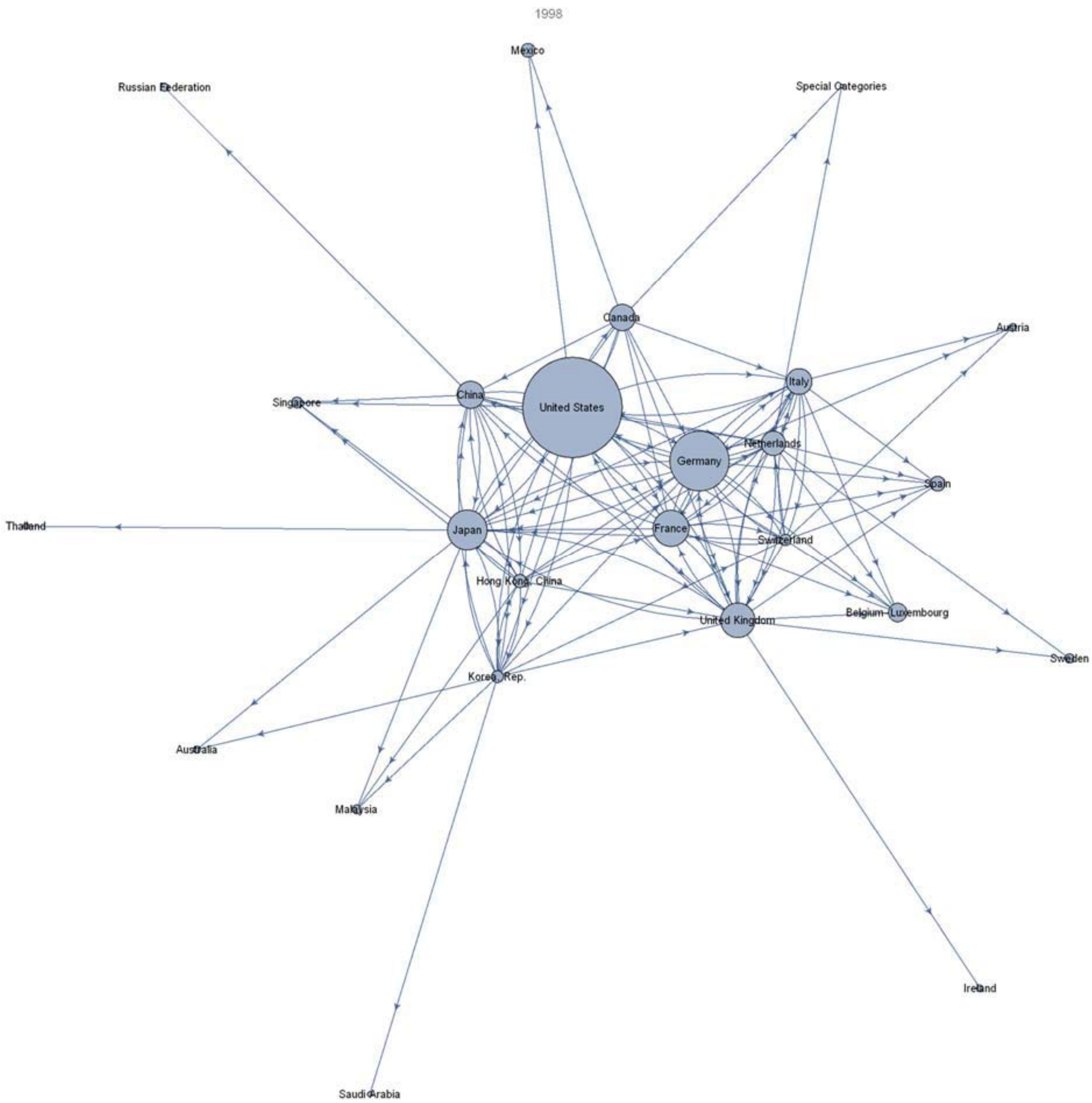


Figure A-3. Trade map in 1998

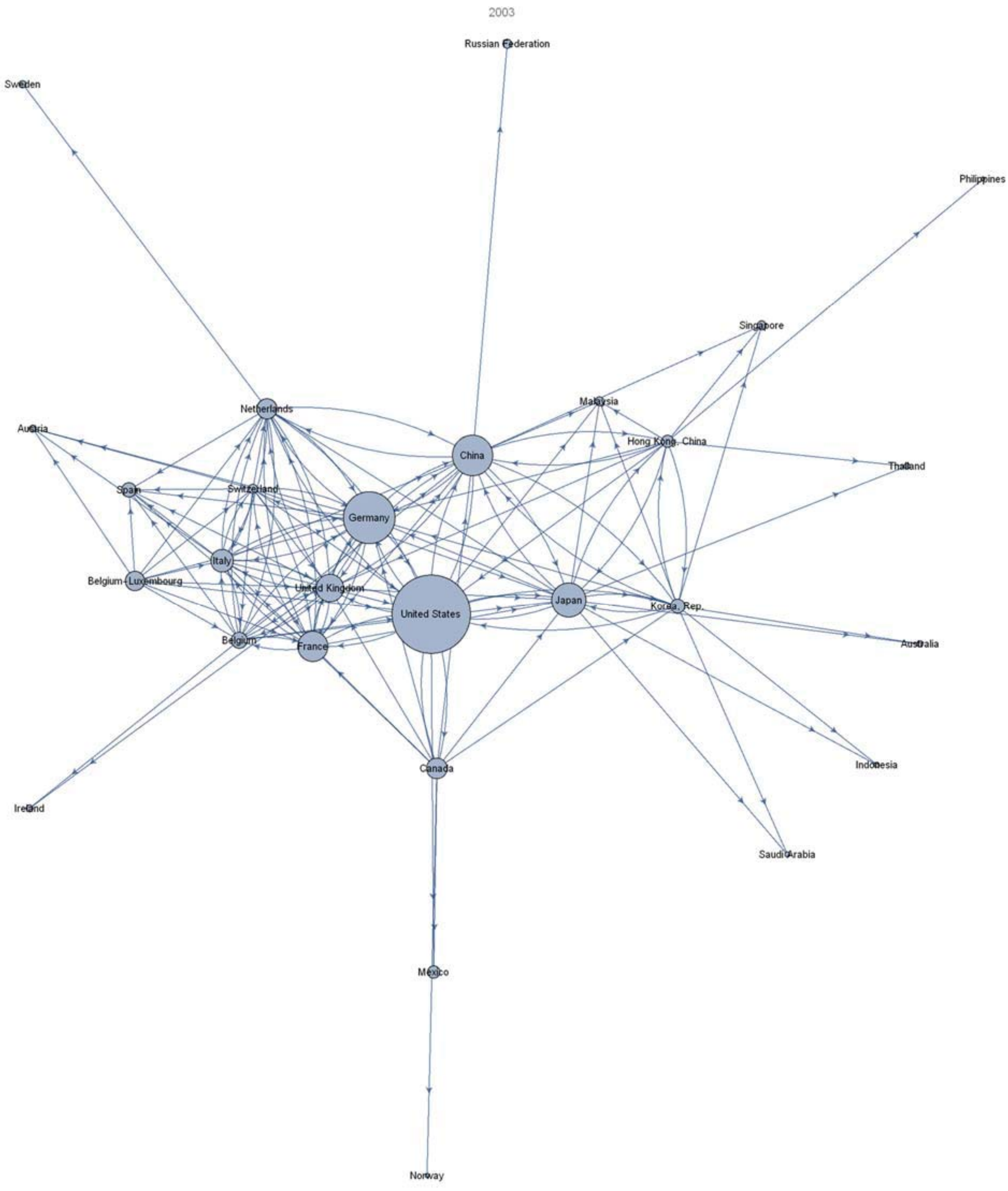


Figure A-4. Trade map in 2003

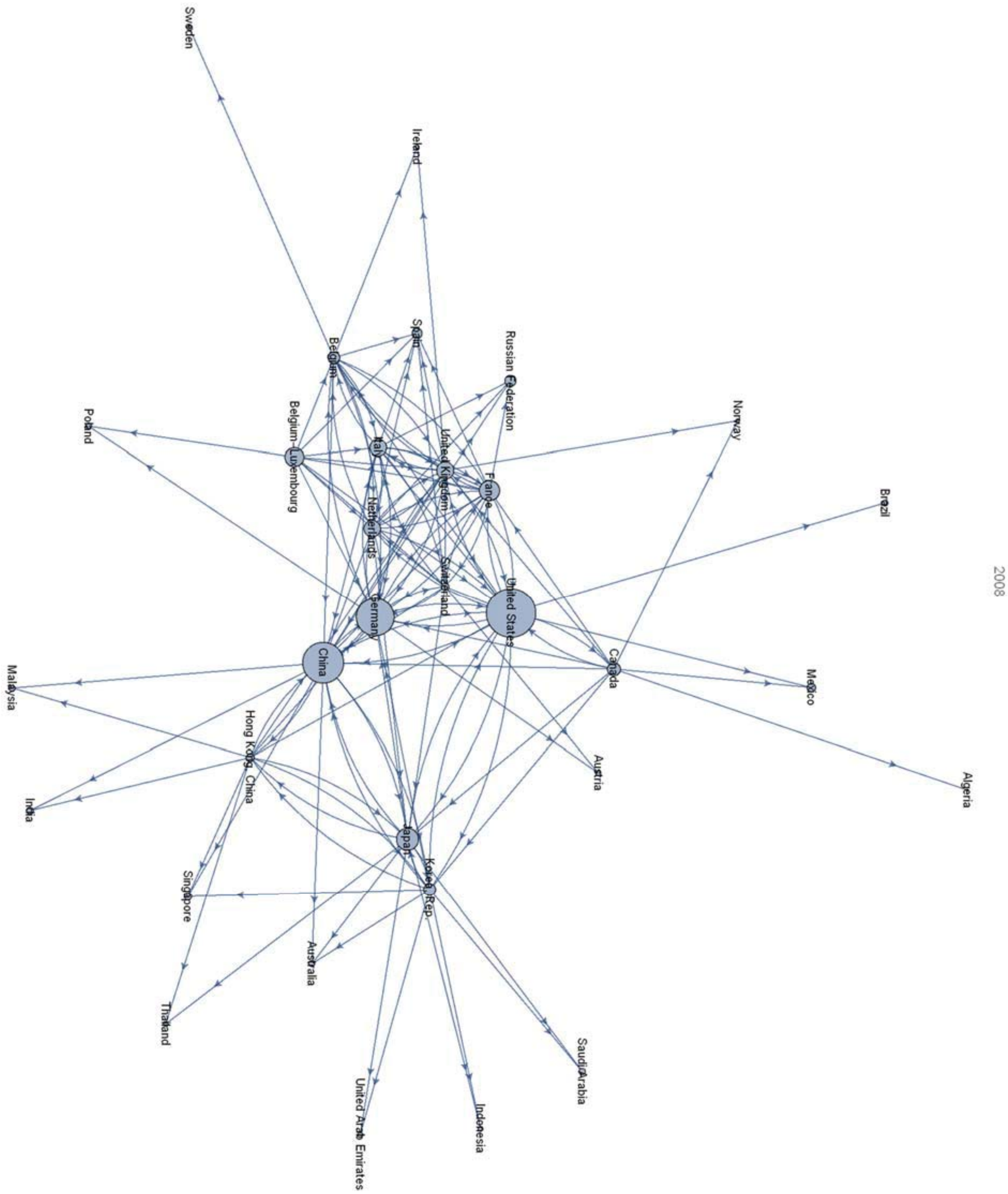


Figure A-5. Trade map in 2008

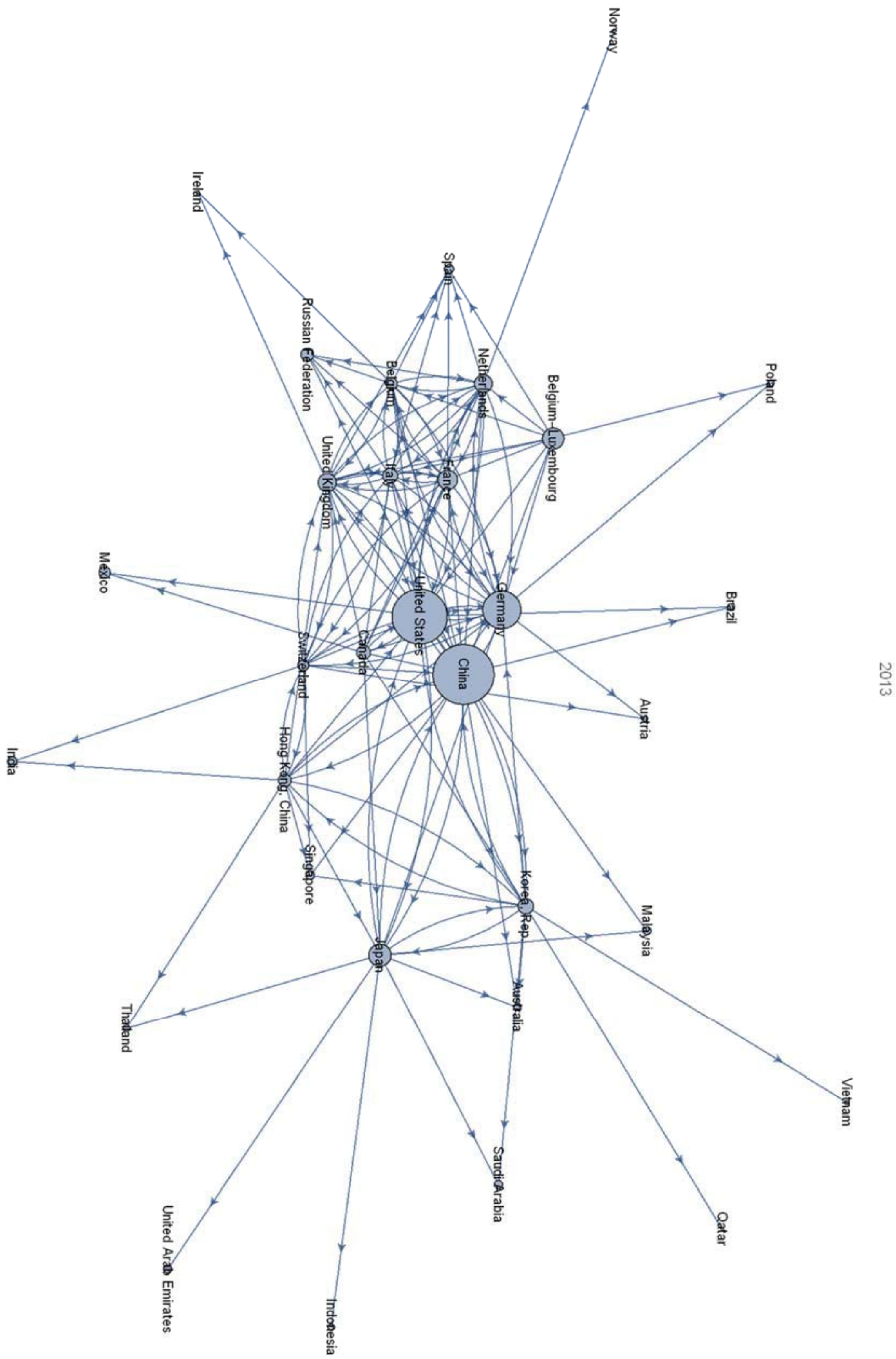


Figure A-6. Trade map in 2013

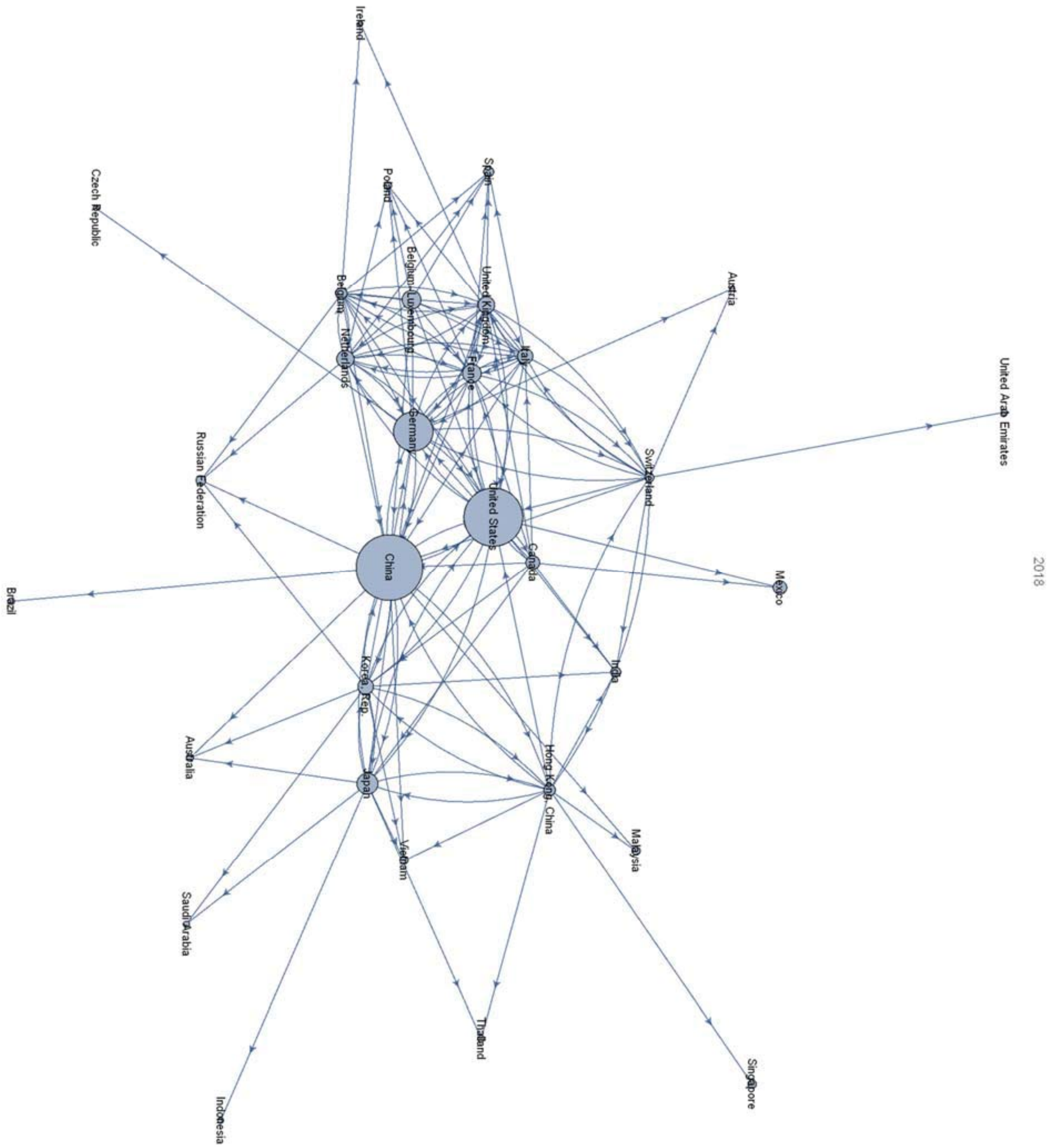


Figure A-7. Trade map in 2018

Source Country		Target Country	average net FDI [T USD]	Source Country		Target Country	average net FDI [T USD]
Australia	->	Canada	1457	United Kingdom	->	Luxembourg	32142
Australia	->	China	3218	United Kingdom	->	United States	20887
Australia	->	France	3372	Ireland	->	France	1787
Australia	->	Japan	10026	Ireland	->	Italy	2009
Australia	->	United States	10388	Israel	->	United States	3414
Austria	->	Germany	1578	Italy	->	Austria	1546
Belgium	->	France	1904	Italy	->	Germany	3873
Belgium	->	United Kingdom	1319	Italy	->	France	20817
Belgium	->	Japan	1495	Italy	->	United Kingdom	2724
Belgium	->	United States	24978	Italy	->	Netherlands	2098
Canada	->	China	2068	Japan	->	Switzerland	2034
Canada	->	Ireland	1215	Japan	->	France	2919
Canada	->	Japan	2034	Luxembourg	->	Canada	12273
Canada	->	Netherlands	2569	Luxembourg	->	Germany	1646
Switzerland	->	Canada	1985	Luxembourg	->	Italy	7521
Switzerland	->	United Kingdom	13669	Luxembourg	->	United States	29943
Chile	->	Canada	1263	Mexico	->	Canada	2665
Chile	->	United Kingdom	1766	Mexico	->	Germany	2309
Chile	->	Italy	1558	Mexico	->	Italy	1017
China	->	Switzerland	2591	Mexico	->	Japan	2248
China	->	Germany	6051	Mexico	->	United States	13644
China	->	Japan	9824	Netherlands	->	France	5520
China	->	Korea	4972	Netherlands	->	United Kingdom	14735
China	->	Netherlands	1325	Poland	->	Germany	2440
China	->	United States	2310	Sweden	->	China	1077
Czech	->	Germany	2472	Sweden	->	Germany	2235
Germany	->	Belgium	1593	Sweden	->	United Kingdom	3883
Germany	->	Switzerland	6597	Sweden	->	Japan	1083
Germany	->	Denmark	1394	United States	->	Austria	1579
Germany	->	Japan	5409	United States	->	Canada	61789
Germany	->	Netherlands	12967	United States	->	Switzerland	17736
Spain	->	Germany	4103	United States	->	Germany	34938
Spain	->	France	3077	United States	->	Spain	7532
France	->	Switzerland	5373	United States	->	France	29510
France	->	Luxembourg	15006	United States	->	Ireland	2064
United Kingdom	->	Canada	6681	United States	->	Italy	1556
United Kingdom	->	Denmark	1404	United States	->	Japan	73245
United Kingdom	->	Spain	4739	United States	->	Korea	6540
United Kingdom	->	Ireland	9538	United States	->	Netherlands	28034
United Kingdom	->	Japan	25252	United States	->	Sweden	1593

Table 1. Average net FDI flows.